4a: [HKK 4.11] Consider the following procedures:

```
(define factorial
  (lambda (n)
    (if (= n 0)
        1
        (* n (factorial (- n 1))))))

(define factorial-sum1 ; returns 1! + 2! + ... + n!
  (lambda (n)
    (if (= n 0)
        0
        (+ (factorial n)
           (factorial-sum1 (- n 1))))))

(define factorial-sum2 ; also returns 1! + 2! + ... + n!
  (lambda (n)
    (define loop
      (lambda (k fact-k addend)
        (if (> k n)
            addend
            (loop (+ k 1)
                  (* fact-k (+ k 1))
                  (+ addend fact-k)))))
    (loop 1 1 0)))
```

In answering the following, assume that n is a nonnegative integer. Also, justify your answers.

(a) Give an exact formula (not using θ-notation) for how many multiplications the procedure `factorial` does as a function of its argument n.

(b) Give an exact formula for how many multiplications the procedure `factorial-sum1` does (implicitly through `factorial`) as a function of its argument n.

(c) Give an exact formula for how many multiplications the procedure `factorial-sum2` does as a function of its argument n.

4b: Where possible, fill in the blanks in the following statements with positive real numbers so as to make the statement true. Where impossible, put a question mark in the blank and explain why it cannot be filled in. In cases which aren't obvious, be sure to show your work.

(a) (Sample) $n-1$ is $\Theta(n)$ because for any $n \geq 2$, we know that $\underline{1/2} \cdot n \leq n-1 \leq \underline{1} \cdot n$.

(b) $3n$ is $\Theta(n)$ because for any $n \geq \underline{\quad}$, we know that $\underline{\quad} \cdot n \leq 3n \leq \underline{\quad} \cdot n$.

(c) $3n-20$ is $\Theta(n)$ because for any $n \geq \underline{\quad}$, we know that $\underline{\quad} \cdot n \leq 3n-20 \leq \underline{\quad} \cdot n$.

(d) n^3+6n^2 is $\Theta(n^3)$ because for any $n \geq \underline{\quad}$, we know that $\underline{\quad} \cdot n^3 \leq n^3+6n^2 \leq \underline{\quad} \cdot n^3$.

(e) n^3+6n^2 is $\Theta(n)$ because for any $n \geq \underline{\quad}$, we know that $\underline{\quad} \cdot n \leq n^3+6n^2 \leq \underline{\quad} \cdot n$.

4c: [HKK 4.13] Consider the following procedure:

```
(define bar
  (lambda (n)
    (cond ((= n 0) 5)
          ((= n 1) 7)
          (else (* n (bar (- n 2)))))))
```

How many multiplications (expressed in Θ notation) will the computation of `(bar n)` do? Justify your answer. You may assume that n is a nonnegative integer.

4d: [HKK 4.14] Consider the following procedures:

```
(define foo
  (lambda (n)        ; computes n! + (n!)^n
    (+ (factorial n) ; that is, (n! plus n! to the nth power)
       (bar n n))))
(define bar
  (lambda (i j)    ; computes (i!)^j (i! to the jth power)
    (if (= j 0)
        1
        (* (factorial i)
           (bar i (- j 1))))))

(define factorial
  (lambda (n)
    (if (= n 0)
        1
        (* n (factorial (- n 1))))))
```

(a) Exactly how many multiplications will `(factorial n)` do?

(b) Exactly how many multiplications will `(bar i j)` do? Be sure to count those done in `bar` as well as those done in `factorial`.

(c) Exactly how many multiplications will `(foo n)` do? Also express your answer using Θ-notation.

(d) How much time will `(foo n)` take? Express your answer using Θ-notation.

How many multiplications (expressed in Θ notation) will the computation of `(foo n)` do? Justify your answer. Be sure to count multiplications done by either of the other procedures.