PHY370 Advanced Mathematical Methods of Physics

Gustavus Adolphus College Spring 2023

Instructor: Dr. Steven Mellema

Office: Olin Hall 210 **Office Hours:** MTWRF 10:30am-11:20pm

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Textbooks:

Mathematical Methods for Scientists and Engineers, by Donald McQuarrie, University Science Books, ©2003

Mathematical Handbook of Formulas and Tables, by Murray R. Spiegel (Schaum's Outline Series)

Course Description and Objectives:

In this course we extend the development of mathematics to topics essential in advanced physics and engineering. The course includes topics in linear algebra, differential equations, Sturm-Liouville theory, and special functions, and explores both analytical and numerical techniques. Practical objectives are:

1. to expose the student to the formalism of the theories and involve them in handling the operational techniques of problem solving;

2. to prepare students for the study of quantum mechanics in their senior year, using Dirac notation to connect both the matrix and eigenfunction representations; and

3. to build upon the mathematical techniques introduced in previous mathematics and physics courses, and to prepare the student for the mathematical level of introductory graduate courses in physics and engineering.

Course Policy and Evaluation:

1. **Class Meetings and Reading Assignments**: The class will meet five days a week from 9:00-9:50AM for lecture, small-group problem solving, homework review and, occasionally, for exams. Attached is a daily calendar of activities for the course. When reading assignments are made for a class session, the reading is expected to be completed **before** coming to the class.

2. **Homework**: Homework problems will be assigned according to topics from the textbook, and are due at the beginning of class on the due date listed on the calendar. Late homework may be accepted at the discretion of the instructor with a reduction in credit of 20% per week.

3. **Use of Computers for Homework**: Occasionally homework problems will be assigned which require a numerical solution. These problems will be specifically assigned as computer problems. For the solutions of these computer problems, you may use any computational tools with which you are familiar, including Mathematica, Python, MATLAB, etc. However, **for non-numerical problems involving**

algebra and calculus, the use of Mathematica or other Computer Algebra/Calculus Systems is forbidden. **No credit for such problems will be given for solutions done by Mathematica.**

4. **Group Problems**: Frequently in class, students will work together, in assigned groups of 3-4 members, to cooperatively solve problems. A group solution will be submitted, with all group members receiving the same grade. There will be no make-up for group problems missed due to absence.

5. **Problem Presentations**: After completing the lectures for each topic in the textbook, we will take one day to have example problem solutions presented to and discussed by the class. These problems will be assigned to specific student presenters approximately one week in advance, and all students will take turns to present problem solutions. Students will earn credit both for their presentations and for their thoughtful discussion of others' presentations.

6. **Attendance** : Regular attendance at all class meetings is expected. Students will be held responsible for informing themselves of all announcements/assignments made in class.

7. **Use of Electronic Devices in Class:** The use of cellular phones, tablets, and laptop computers during the lectures is prohibited. Exceptions may be made to accommodate student accessibility.

8. **Exams :** There will be four hour exams and a two-hour final exam (see the calendar below). Students must arrange **in advance** to take an exam at other than the scheduled time, and may do so **only** for a valid health or school-related reason. (It is the responsibility of the student to inform the instructor during the first week of the semester regarding any anticipated absences due to required field trips, athletic events, musical performances, or other extra-curricular activities.) Exams missed without pre-arrangement are entered as zero credit and cannot be made up.

9. **Evaluation** :

Homework	25%
Group Problems	10%
In-Class Problem Presentations	10%
Hour Exams	10% each
Final Exam	15%

Assignment of final letter grades will be based upon the following guidelines:

	B+ = 86-90%	C+ = 74-78%	D+ = 62-66%
A = 94-100%	B = 82-86%	C = 70-74%	D = 58-62%
A- = 90-94%	B- = 78-82%	C- = 66-70%	

10. **Incompletes** : A grade of incomplete will **only** be given for work not completed due to circumstances beyond the control of the student (*this is the College policy*).

11. **Academic Honesty**: Having signed and agreed to abide by the College's Honor Code, students thereby pledge that, in all academic exercises and examinations, they shall submit their own work. In the context of this course, students are expected to collaborate and to discuss their out-of-class assignments. However, submitting under one's own name work that is merely copied from another is a violation of the Honor Code. Furthermore, seeking outside assistance during exams is expressly

forbidden. A full description of the Academic Honesty Policy and the Honor Code can be found in the Academic Catalog (online at:www.gustavus.edu/general_catalog/current/acainfo).

12. **Requesting Accommodations**: Gustavus Adolphus College is committed to ensuring equitable and inclusive learning environments for all students. If you have a disability and anticipate or experience barriers to equal access, please speak with the accessibility resources staff about your needs. A disability may include mental health, attentional, learning, chronic health, sensory, physical, and/or short-term conditions. Students with a documented elevated risk of COVID-19 may also request academic accommodations. Accommodations cannot be made retroactively; therefore, to maximize your academic success at Gustavus, please contact them as early as possible. Accessibility resources staff are located in the Academic Support Center (https://gustavus.edu/asc/accessibility/) (x7138). Accessibility Resources Coordinator, Corrie Odland, (codland@gustavus.edu), can provide further information.

13. **Help for Multilingual Students:** Some Gusties may have grown up speaking a language (or languages) other than English at home. If so, we refer to you as "multilingual." Your multilingual background is an incredible resource for you, and for our campus, but it can come with some challenges. You can find support through the Center for International and Cultural Education's (https://gustavus.edu/cice/) Multilingual and Intercultural Program Coordinator (MIPC), Pam Pearson (ppearson@gustavus.edu). Pam can meet individually for tutoring in writing, consulting about specific assignments, and helping students connect with the College's support systems. If you want help with a specific task (for example, reading word problems on an exam quickly enough or revising grammar in essays), let your professor and Pam know as soon as possible. In addition, the Writing Center (https://gustavus.edu/writingcenter/) offers tutoring from peers (some of whom are themselves multilingual) who can help you do your best writing.

FEBRUARY2023

subject: Advanced Math Methods period: 2

	MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY	SAT/SUN
			1	2	3	4/5
WEEK 1	6 Classes begin; Syllabus and Introduction to Infinite Series Read: Sections 2.1-2.3	7 Alternating Series Sections 2.4-2.5	8 Power Series Sections 2.6-2.8	9 The Gamma and Beta Functions Sections 3.1-3.2	10 The Error Function and Elliptic Integrals Sections 3.3, 3.5	11/12
WEEK 2	13 The Dirac Delta Function Read: Section 3.6	14 Problem Presentations: Chapters 2&3	15 Determinants Chapters 2-3 Homework Due Section 9.1	16 Gaussian Elimination Section 9.2	17 Matrices Section 9.3	18/19
WEEK 3	20 The Rank of a Matrix Read: Section 9.4	21 Vector Spaces Sections 9.5-9.7	22 Problem Presentations: Chapter 9	23 Transformations Chapter 9 Homework Due Section 10.1	24 Eigenvalues and Eigenvectors Section 10.2	25/26
WEEK 4	27 Applied Eigenvalue Problems Read: Section 10.3	28 **Hour Exam #1 (Ch. 2,3,9)**				

MARCH2023

subject: Advanced Math Methods period: 2

	MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY	SAT/SUN
WEEK 4			1 Change of Basis	2 Matrix Diagonalization	3 Quadratic Forms	4/5
	Read:		Section 10.4	Section 10.5	Section 10.6	
WEEK 5	6 Problem Presentations: Chapter 10	7 First-Order ODE's Chapter 10 Homework due	8 First-Order ODE's	9 Second-Order ODE's with Constant Coefficients	10 Systems of Linear Differential Equations	11/12
	Read:	Section 11.1	Section 11.2	Section 11.3-11.4	Section 11.6	
WEEK 6	13 Numerical Solutions to ODE's	14 Problem Presentations: Chapter 11	15 Frobenius' Method Chapter 11 Homework due	16 Legendre's Equation	17 Singularities	18/19
	Read:		Sections 12.1-12.2	Section 12.3	Section 12.4	
WEEK 7	20 Bessel's Equation	21 Bessel Functions	22 Problem Presentations: Chapter 12	23 Phase Plane Chapter 12 Homework due	24 Critical Points	25/26
	Read: Section 12.5	Section 12.6		Section 13.1	Section 13.2	
WEEK 8	27 Stability of Critical Points	28 **Hour Exam #2 (Ch. 10-12)**	29 Nonlinear Oscillators	30 Population Dynamics	31 Problem Presentations: Chapter 13	
	Read: Section 13.3		Section 13.4	Section 13.5		

APRIL2023

subject: Advanced Math Methods period: 2

	MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY	SAT/SUN
						1/2
	..					
	3 **No Class: Spring Break**	4 **No Class: Spring Break**	5 **No Class: Spring Break**	6 **No Class: Spring Break**	7 **No Class: Spring Break**	8/9
WEEK 9	10 Legendre Polynomials Chapter 13 Homework due	11 Orthogonal Polynomials in General	12 Sturm-Liouville Theory	13 Eigenfunction Expansions	14 Green's Functions Techniques	15/16
	Read: Section 14.1	Section 14.2	Section 14.3	Section 14.4	Section 14.5	
WEEK 10	17 Problem Presentations: Chapter 14	18 Fourier Series Chapter 14 Homework due	19 Fourier Sine/Cosine Series	20 Convergence of Fourier Series	21 Fourier Series and ODE's	22/23
	Read:	Section 15.1	Section 15.2	Section 15.3	Section 15.4	
WEEK 11	24 Problem Presentations: Chapter 15	25 Partial Differential Equations Chapter 15 Homework due	26 Laplace's Equation	27 The One-Dimensional Wave Equation	28 Two-Dimensional Wave Equation	29/30
	Read:	Section 16.1	Section 16.2	Section 16.3	Section 16.4	

MAY2023

subject: Advanced Math Methods period: 2

	MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY	SAT/SUN
WEEK 12	1 **Hour Exam #3 (Ch. 13-15)** Read:	2 The Heat Equation Section 16.5	3 The Schrödinger Equation Section 16.6	4 Problem Presentations: Chapter 16	5 Laplace Transforms Chapter 16 Homework due Section 17.1	6/7
WEEK 13	8 Laplace Inverse Transforms Read Section 17.2	9 Laplace Transforms and ODE's Section 17.3	10 Laplace Transforms and PDE's Section 17.4	11 Fourier Transforms Section 17.5	12 Fourier Transforms and PDE's Section 17.6	13/14
WEEK 14	15 Problem Presentations: Chapter 17 Read	16 Exam Review Chapter 17 Homework due	17 **Hour Exam #4 (Ch. 16-17)**	18 Final Exam Review	19 **No Class: Reading Day**	20/21
	22	23 **Final Exam: 10:30am-12:30pm**	24	25	26	27/28
	29	30	31			