Meeting the Challenge of Mathematics Reform for Students with LD

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The purpose of this article was to examine the sustained effort to reform K through 12 mathematics instruction in this country over the last 10 years and the implications of this reform for students with learning disabilities. We begin by describing three forces that have driven mathematics reform: shifting theoretical paradigms, disappointing levels of mathematics performance of students in the United States, and the impact of rapidly changing technologies. We then discuss concerns about this reform from the special education community. In the second half of the article, we provide synthesized special education research relevant to mathematics reform, along with thoughts about future directions in mathematics education for students with learning disabilities.

One of the major problems with the 1989 Standards was that they did not specify how the call for uniformly high outcomes for students at a national level would be reconciled with the varying needs inherent in serving a large and diverse population of students. This problem was acutely apparent to the special education community, as students with disabilities were neglected throughout the document. There was virtually no mention in the 1989 Standards of how the suggested curricular and pedagogical reforms would affect students with disabilities, the majority of whom were or would be receiving mathematics instruction in general education mathematics classes. At a practical level, both special and general educators were concerned about how these major shifts in curriculum and teaching methods would occur, given the limited resources available to enact them.

Despite the document’s vagueness about implementation and diversity, there was enthusiastic support for the reform across many groups in the 1990s, particularly among policymakers. States began investing considerable money and effort in curriculum development and teacher in-service to facilitate acquisition of the necessary pedagogy to implement a new curriculum. By 1993, 41 states had taken action to adopt the 1989 NCTM Standards. As of 2001, all but one state had new content or curriculum standards in mathematics.

To probe current thinking about mathematics reform and its implications for students with learning disabilities, it is important to go beyond the Standards themselves. Too often, broad criticism of the Standards, much of which is reviewed in this article, belies a specific concern about pedagogical meth-
ods. Disentangling some of the different forces behind the Standards helps elucidate the promise and the problems special educators see in contemporary math reform.

In the first half of the article, we examine three of the more important forces that have shaped mathematics reform over the last 30 years: shifts in theoretical paradigms, disappointing levels of mathematical performance of students in this country, and broad advancements in technology. We then provide an analysis of special education’s response to the Standards over the last decade. In the second half of the article, we describe current special education research and promising practices that are in keeping with the 2000 NCTM Standards and should facilitate mathematics instruction for students with learning disabilities in the context of curricular and pedagogical reform.

**Forces Shaping Mathematics Reform**

**Shifts in Theoretical Paradigms**

Major changes in theories of learning have been as apparent in mathematics education as they have been in other disciplines. Behaviorism was the prevailing theory of learning and instruction during the 1950s and 1960s. When cognitive psychology reemerged in the late 1960s and early 1970s, researchers in both general and special education began to consider cognitive explanations of learning and incorporate these notions into their research programs. Cognitive psychology departed from the strict orthodoxy of behaviorism and, with its several interrelated branches, began to affect educational research and practice. Neo-Plagietians (e.g., Case, 1985; Flavell, Miller, & Miller, 1993), information processing theorists (e.g., Simon, 1980; Sternberg, 1985), and sociocultural theory (e.g., Vygotsky, 1989; Wertsch, 1991) continue to be major influences on research and practice. Intervention researchers, however, recognized the utility of principles like task analysis and therefore did not abandon behaviorism entirely. This trend toward eclecticism in theoretical foundations for designing instruction was especially apparent in special education (Gersten & Baker, 1998; Swanson, 1988, 1989).

The NCTM Standards are grounded in cognitive and constructivist approaches to learning, as reflected in their emphasis on developing children’s ability to think about mathematics. In order to accomplish this, the Standards advocate numerous changes in content; pedagogy; learning tasks and experiences; and the context for learning, curriculum, and evaluation. Six principles guide educators in their attempts to reform mathematics instruction:

1. equity, which implies high expectations and instructional support for all students;
2. a well-articulated and coordinated curriculum
3. effective teaching that demonstrates an understanding of students’ knowledge and then pro-
vides challenging activities to encourage new learning;
4. learning, which implies the active construction of mathematical knowledge by students;
5. assessment that supports learning and is useful to students and teachers; and
6. technology, as essential in mathematical instruction and learning.

At the core of the constructivist agenda are assertions that learning is an active, social, and interactive process; that learners construct an understanding of subject matter rather than copying it directly from the teacher; and that authentic activities are the foundation for the construction of knowledge (Bransford, Brown, & Cocking, 1999; Hiebert et al., 1997). This approach considers learners’ misunderstandings or misconceptions to be important learning opportunities. As fundamental as these tenets may appear, over the last 20 years various positions on constructivism have emerged. This has complicated a clear understanding of this theoretical approach to instruction, leaving it ill-defined in the minds of many (Prawat, 1999). One can find constructivist literature that focuses predominantly on the individual (Von Glaserfeld, 1991), on the social or sociocultural (Forman, Minick, & Stone, 1993; Palincsar, 1998), and even on the political or “emancipatory” (Vadeboncoeur, 1997).

Both the individual and the social perspectives are widely apparent in the general education mathematics literature. Von Glaserfeld’s (1995) radical constructivism, for example, places a high value on individual constructions or inventions of mathematical knowledge. He suggested that there are limits to how much shared meaning exists between students about a given concept and that a student’s misconception cannot simply be replaced by the teacher’s “right way of looking at things.”

In contrast, the social constructivist perspective highlights classroom community and discourse. Making common mathematical topics, such as two-digit addition or subtraction, problematic and treating them as “ill-defined” are frequently discussed in this strand of the reform literature as a means to get students to “think mathematically” (Ball, 1993; Cobb, Wood, & Yackel, 1993; Hiebert et al., 1997; Lampert, 1990; Lampert, Rittenhouse, & Crumbaugh, 1996; Resnick, 1988). Geary (1994) noted that this view, if taken to an extreme, makes mathematics almost entirely a social enterprise in which discussion and disagreement are the basis for a student’s conceptualization and reconceptualization of key concepts.

Social constructivist thinking can also be found in studies that emanate from cultural psychology. Research into the mathematical practices of street vendors (Nunes, Schliemann, & Carraher, 1993), tribal groups (Saxe, 1992), and grocery store shoppers (Lave, 1988) offers a sociocultural, or “situated cognition,” perspective. One theme from this ethnomathematical literature is that even though individuals may perform poorly on standardized or formal measures of mathematical
understanding, they can demonstrate high levels of competence in practical, everyday settings. Another, more obvious message from the ethnomathematical literature is that an individual’s culture can have a profound effect on the development of her or his informal mathematics knowledge.

The striking variations among these views of constructivism make a single account of the theory problematic. As Richardson (1997) observed, constructivism does not offer one simple perspective on teaching. For these reasons, and because special education is so grounded in behavioral theory, many special educators have understandably responded with caution to the shift toward cognitive and constructivist views of teaching and learning (e.g., Gersten & Baker, 1998; Harris & Graham, 1994, 1996; Mercer, Jordan, & Miller, 1994).

Special education research has a history of placing a considerable emphasis on rote learning and mastery of math facts and algorithms for basic operations (e.g., addition, multiplication) and limiting instruction in problem solving (see Swanson, Hoskyn, & Lee, 1999). Several researchers have noted that even though traditional methods, such as direct instruction, may be effective for teaching factual content, “there is less evidence that this instruction transfers to higher order cognitive skills such as reasoning and problem solving” (Palincsar, 1998, p. 347). Given the long-standing need to provide general education curriculum for students with disabilities, special educators must reconsider traditional approaches and provide instruction that is more consistent with the reform agenda.

Disappointing Levels of Mathematical Performance

General education also has been described as overemphasizing rote instruction and didactic teaching. Criticism of the highly procedural nature of mathematics instruction in the United States preceded the “new math” efforts of the 1960s. An array of textbooks, commentaries, and research from the 1940s and 1950s (e.g., Brownell, 1945; Marks, Puri, & Kinn, 1958; Wertheimer, 1959) critiqued instruction that focused only on rote mastery of arithmetic knowledge. Similar criticism of instruction in mathematical concepts and problem solving appeared in the 1980s (see Grouws, 1992). The basis of the argument was that instruction was unsatisfactory because it concentrated almost exclusively on the mastery of algorithms used to perform operations on whole and rational numbers. As a result, students failed to achieve a sufficient conceptual understanding of the core concepts that underlie those operations and algorithms (Baroody & Hume, 1991; Hiebert, 1986; Hiebert & Behr, 1988; Skemp, 1987).

Another concern was that problem-solving instruction typically involved only problems that could be solved in “five minutes or less” (e.g., Doyle, 1988; Schoenfeld, 1988). This type of rapid and artificial problem solving is most apparent when students are taught to search for key words (e.g., “more” or “gave away”) or use a single strategy (e.g., “make a table”) to solve a set of predictable or highly structured problems. Such an approach may enable students to complete what has been called “end of the chapter problems.” However, critics argued that these methods do little to foster a deeper sense of problem solving. In fact, they may lead many students to give up when faced with more complex problems that require extended effort or several incorrect attempts before a correct solution is achieved. Yet, complex problems reflect the kind of effort required in authentic problem solving and more sophisticated mathematics.

Results from international studies of students’ mathematical proficiency over the last two decades have been disappointing for the United States, thus reinforcing educators’ concerns. The most recent studies (e.g., Third International Mathematics and Science Study [TIMSS]; Beatty, 1997) and subsequent TIMSS-Repeat (U.S. Department of Education, National Center for Education Statistics, 2000), each involving about 40 countries, confirmed that students in the United States are not performing as well as students in many other developed countries, especially those in Asia. In 1999, U.S. students ranked about in the middle compared to students from 38 other countries. The top-performing countries were Singapore, Republic of Korea, Chinese Taipei, Hong Kong, Japan, and Belgium. As Schmidt and his colleagues (1997) suggested, mathematics instruction in the United States suffers from a “splintered vision,” with curricula that focus on too many superficially taught topics in a given year. More successful approaches, found particularly in Asian countries, tend to focus on fewer topics. The lessons are often devoted to the analysis of a few examples, and teachers encourage students to share different solutions to problems (Office of Educational Research and Improvement, 1998; Stevenson & Stigler, 1992; Stigler & Hiebert, 1999). Most certainly, there are educators (e.g., Berliner & Biddle, 1995; Bracey, 1997) who disagree with the thrust of this criticism; however, their commentaries tend to address long-standing and broad-based critiques of U.S. education rather than problems with mathematics education per se.

Technological Advances

The broad impact of technological advancements over the past 30 years has been startling and well documented. Much has been made of our current economic transformation from an industrial society to an information economy (Davis & Maher, 1996; Greider, 1997; Reich, 1991). It is ironic, however, that with constant technological change all around us, it is difficult to appreciate either the short- or long-term effects of these changes. In computing, microprocessing power has doubled every 18 to 24 months since the late 1950s (i.e., Moore’s Law) and is expected to continue well into the first two decades of the 21st century (Patterson, 1997). As it affects desktop computers, the doubling of processing power up to this point in time has been impressive but not overwhelming. Continued doubling in processing power over the next 20 years coupled
with declining costs will create at frequent intervals personal desktop computing systems of entirely different magnitudes than we have ever experienced. Over the next decade, personal computers will approach supercomputing capacities.

At the other end of the spectrum, we are entering a profoundly important era of embedded computing. The term embedded computing refers to an ever-growing array of inexpensive computing devices found in everyday appliances and tools. No longer will handheld calculators present the “revolutionary” promise for mathematics instruction, as Romberg (1992) predicted. Rather, a variety of handheld devices that perform important computational and communicative functions will become increasingly available. The range of computing tools, from handheld devices to powerful computers, will increasingly be used to perform the very computational functions that students have practiced on a daily basis by hand. These tools already allow today’s students (and workers) to operate on data at a much higher level (e.g., analyze it for trends, perform statistical operations, solve complex problems).

These technological trends have considerable implications for students with learning disabilities. For students who continue to learn a narrow range of mathematics (e.g., instruction that focuses largely on paper-and-pencil mastery of computational problems), this will be a growing problem. Continued advances in technology will only accentuate the gap between what is typically taught to students with learning disabilities and what individuals need to know in a world filled with computing devices (see Goldman, Hasselbring, & the Cognition and Technology Group at Vanderbilt, 1997).

**Special Education, Mathematics Reform, and the NCTM Standards**

The first source of criticism of mathematics reform is the shift from traditional didactic models of instruction to constructivism. Schools are increasingly adopting mathematics programs such as Everyday Mathematics and the Connected Mathematics Program that embrace constructivist approaches. To exemplify, Tarver (1996) argued that constructivism is tantamount to discovery learning and, as a pedagogical approach, most likely will lead to even greater failure for students with learning disabilities. She ardently defended direct instruction as the most effective approach to education for these students. Her position stands as a marked counterpoint to the trend toward full inclusion for students with disabilities, IDEA’s requirement to provide access to the general education curriculum for these students, and the increasing emphasis on developing students’ mathematical “power.”

In contrast, others have argued that traditional special education instructional methods and constructivism are more compatible than not. They have made a conscious effort to distinguish between the kind of constructivism that stresses child-determined, guided discovery and a more structured vari-

Even Von Glaserfeld (1991), who is regarded by some as having the most extreme epistemology views on constructivism, acknowledged that memorization and rote learning are unavoidable in education. Special educators’ efforts to ensure a place for skill instruction in a constructivist framework are both important and understandable when faced with the educational needs of students.

Recently, Gersten and Baker (1998) attempted to merge constructivist and behavioral models of instruction in their effort to link direct instruction with situated cognition. They suggested that behavioral methods like direct instruction provide the necessary skill base for problem solving and are easily incorporated into routines and approaches such as anchored instruction. Botte and Hasselbring’s (1993 study controlled for students’ prior knowledge using a direct instruction math videodisc program to develop a common knowledge base in fractions before implementing the intervention. Based on this preteaching, they were able to contrast the effects of anchored instruction problems with more traditional problem-solving methods and avoid important confounds. A more accurate view of Botte and Hasselbring’s anchored instruction research, at least as it applies to practitioners, is that skill development should be embedded in instruction as needed (Goldman et al., 1997; T. S. Hasselbring, personal communication, October 21, 1999). Reid (1998) made a similar observation in her discussion of how some special educators misinterpret scaffolding and its application in practice. Concepts such as scaffolding, situated cognition, and direct instruction emanate from different theories of learning, motivation, and instruction (see Greeno, Collins, & Resnick, 1996).

Pedagogically, it is difficult to see how a teacher would provide teacher-directed, step-by-step skill instruction for a lengthy period of time and then shift frameworks and employ interactive scaffolded instruction, inquiry about misconceptions, and authentic tasks, as Gersten and Baker (1998) suggested. Extended hierarchical skill instruction (i.e., instruction that places little emphasis on conceptual understanding) followed by anchored problem solving in an area such as fractions is conceptually problematic. Woodward, Baxter, and Robinson’s (1999) recent study of direct instruc-
tion methods in decimals and percentages delineated the problems of trying to provide a "skills-only" foundation in a mathematical topic before moving on to conceptual understanding or problem solving. They found that students who were taught using direct instruction methods for 4 weeks were not prepared for conceptually based instruction in the subject. A more disturbing finding from this study was that retention of the skills mastered during the instructional intervention quickly atrophied.

The second main source of criticism from special educators has to do with the NCTM Standards themselves. The NCTM Standards have been described as elitist (Hofmeister, 1993), too difficult to implement (Carnine, Jitendra, & Silbert, 1997), devoid of a research foundation (Carnine, 1992; Carnine, Jones, & Dixon, 1994; Kameenui, Chard, & Carnine, 1996; Stein & Carnine, 1999), and representative of discovery-oriented constructivism (Merrill et al., 1994). Even though much of the criticism is polemical, important concerns and common misconceptions about the NCTM Standards are apparent. For example, some of the difficulty special educators have with the NCTM Standards may reside in the way they are written. The NCTM Standards are not intended as a detailed series of objectives for daily instruction nor as an academic research document. Rather, they are written for a broad audience of practitioners, policymakers, and those interested in educational reform. In this regard, they are in keeping with previous publications for diverse audiences (e.g., Everybody Counts [National Research Council, 1989]).

The claim that the NCTM Standards lack a research foundation appears extraordinary on the surface, given research syntheses such as Grouws's (1992) Handbook of Research on Mathematics Teaching and Learning, as well as many other publications over the last two decades (see De Corte et al., 1996). At a more subtle level, however, this concern over the presumed lack of adequate research support generates a more substantive question, one that is typically unstated in the critics' polemics. Hiebert (1999) offered a thoughtful answer to this question in his recent discussion of the relationship between research and standards in any field. He observed that research cannot begin to account fully for all aspects of standards for a field. One central reason for this is that research support is likely to be uneven when standards are developed. Standards, after all, are policy documents. They are intended to reflect what is already known, as well as to promote further research in a given area. Second, and perhaps most important, standards reflect what is valued in education. This observation is crucial in any consideration of mathematics instruction for students with learning disabilities. Narrow attempts to define what "counts" as valid research (e.g., Dixon, Carnine, Lee, Wallin, & Chard, 1998) miss these two points. Putting aside the dubious quality of Dixon et al.'s report (see Becker & Jacob, 2000), it relies on research design criteria that result in an atheoretical collection of studies that offer little, if any, direction for math education. They do little to achieve the kind of convergent understanding needed to move a discipline forward. This view is reinforced throughout the Handbook of Research Design in Mathematics and Science (Kelly & Lesh, 2000). Addressing what is valued in education is a relatively recent goal for those concerned with curriculum design and classroom pedagogy (National Research Council, 2001). As Greeno et al. (1996) claimed, there is an all too common tendency to take what generally appears in commercial materials and find ways to teach that content to students more efficiently, rather than addressing more basic questions of what is worth knowing.

Reformers like Hiebert (1999) also noted that the traditional methods for teaching mathematics in the United States have done little to help us meet the kind of rigorous outcomes described in the NCTM Standards, nor have they helped improve standings on international tests such as the TIMSS. Stigler and Hiebert's (1999) analysis of U.S., German, and Japanese curricula and classroom pedagogy detailed the problems with process-product approaches to mathematics in which students concentrate on a mastery of procedures, with only a marginal focus on conceptual understanding. As mentioned earlier, these observations have direct implications for intervention research in special education. Specifically, they argue that protracted instruction in skills and procedures without a balanced emphasis on conceptual understanding and problem solving is problematic.

In summary, the two main sources of concern for special educators (i.e., constructivism and the lack of a research base for the NCTM Standards) are much more complex than commonly thought. Undoubtedly, many in the field will always find constructivism troubling, even with a well-articulated skills component. This is understandable, given the disconnection between constructivism and an "embedded skills" approach to mathematics, which is new and rarely articulated for students with learning disabilities. Nonetheless, it should be clear that the two other forces guiding math reform—the long-standing critique of mathematics and the continued evolution of technology—argue for a serious reexamination of common mathematics practices for students with learning disabilities.

NCTM Standards and Instruction for Students with LD

This section describes new directions in mathematics research that seem to align with the NCTM Standards and also appear to hold particular promise for improving performance in math fact acquisition, computation, and problem solving for students with learning disabilities. Some of these directions have a substantial research base, and others are emerging but hold promise for future research.

It should be noted that this section is not an exhaustive attempt to discuss how mathematics topics should be adapted for students with learning disabilities. In fact, there are many areas of mathematics (e.g., measurement, geometry, simple
levels of probability and statistics) about which there is little
research involving students with learning disabilities. The
relative dearth of research in mathematics for students with
learning disabilities is apparent in recent meta-analyses of
intervention research (see Swanson et al., 1999). This section
also does not attempt to describe how students with learning
disabilities should be taught mathematics using a small set of
generic principles of curriculum design or pedagogy. We feel
that approach understates the subtle but critical ways in which
content issues in a domain guide instruction.

One constraint that must be mentioned in any discussion
of improving practice for students with learning disabilities is
the issue of limited instructional time. Time constraints have
an impact on what, how much, and when students learn. Too
often in special education, students are shortchanged in
mathematics instruction, particularly higher level mathematics,
because they have so many other pressing needs, including, but
not limited to, academics. Such time constraints force us to
examine carefully what we want students to learn and how
this will affect educational outcomes. Rather than developing
a more efficient reworking of traditional mathematics using a
common hierarchy of skills, outcomes need to be considered
in light of promising directions in recent mathematics re-
search (see Goldman et al., 1997, for a further discussion of
this issue).

**Instruction in Math Facts**

Information processing approaches to math instruction for
students with learning disabilities emphasize the importance
of fluency in fact retrieval. The argument is that quick and ef-
icient math fact recall, or automaticity, enables students to
devote more of their cognitive resources to the procedural
knowledge associated with learning algorithms (Gerber, Sem-
mel, & Semmel, 1994; Pellegrino & Goldman, 1987). Several
studies suggest that students with learning disabilities tend to
use immature strategies when they learn math facts. Goldman,
Pellegrino, and Mertz (1988), for example, found that students
with learning disabilities were well behind their peers in using
**min** counting (e.g., adding 2 + 9 as 9 + 2) and direct retrieval
strategies for addition facts. Students with learning disabilities
tended to use **counting all** strategies (i.e., laborious one-by-
one counting to achieve answers) even after extended practice.

Putnam, deBettencourt, and Linhardt (1990) found a
similar tendency for immaturity strategies when students with
learning disabilities were taught to use derived fact strategies
for retrieving facts, such as doubling (e.g., 6 + 7 = 6 + 6 + 1),
sharing (7 + 9 = 8 + 8), and going through 10 (9 + 4 = 10 + 3).
Although this work focused less on automaticity than Gold-
man et al.'s (1988) article and more on part–whole conceptual
understanding, both researchers concluded that the perfor-
mance of students with learning disabilities was **delayed**
and not cognitively different from that of normally achieving chil-

Research suggests that at least a subset of remedial stu-
dents and those students with disabilities (specifically, math
dyscalculia) have immature strategies such as counting all, but
their actual difficulties are much more profound. Geary’s
(1994) summary of a number of related studies on this issue
focused on two concerns. First, his research with first grade
students, in particular, found that about half of the children were misiden-
tified for remedial and special education services and did not
show any form of a cognitive deficit (Geary, 1990; Geary,
Bow-Thomas, & Yao, 1992). The remainder of students, how-
ever, had significant problems representing facts in long-term
memory and were highly inconsistent in the speed at which
they retrieved these facts. For these children, Geary argued,
retrieval is not likely to improve, at least not without extensive
remedial training. In his view, students with learning dis-
abilities are categorically different from those who exhibit
developmental delays. A comprehensive accounting of the
difficulties students with developmental delays face in learn-
ing math facts is complicated by the structure of facts them-

Dehaene (1997) observed that the complex network of
associations for multiplication facts in conjunction with less
practice on some of the more complex facts (e.g., 9 × 6, 7 × 8)
leads to confusion and recall errors. Research-based tech-
niques for teaching automaticity are rooted in information
processing theory, which suggests methods that have general
application for teaching a wide array of declarative knowl-
dge. Hasselbring, Goin, and Bransford’s (1988) methods, for
example, delineate the importance of testing students, in-
roducing new facts in small steps, providing systematic review,
and controlling response time as ways of moving students from
counting all strategies to direct retrieval. Others (e.g., Cybri-

Jones, Thomson, and Toohey (1985) demonstrated the
potential for developing number sense in students with learning
disabilities by using mathematical strategies such as doubling, shari-
ing, and going through 10. For example, students are taught
that addition problems such as 8 + 5 can be seen as 8 + 2 + 3
or 10 + 3. This is one example of a math fact strategy that de-
velops number sense in children and potentially has a broad application in mathematics learning. Woodward and Eckholds (1999) examined the extension of facts into number sense in a year-long study of academically low-achieving first graders. These students had initial difficulty extending their knowledge of simple addition facts to larger quantities (e.g., from $3 + 2 = 5$ to $30 + 20 = 50$). However, after sustained practice, they showed dramatic improvement in this skill and also improved their understanding of the principle of commutation in addition.

**Instruction in Computations**

Teaching students traditional algorithms for performing the four basic operations in mathematics (e.g., regrouping in subtraction, dividing decimal numbers) is one of the most common instructional activities for students with learning disabilities. Observations of special education classrooms indicate that this type of skill instruction dominates (Parmar & Cawley, 1991, 1995; Rieth, Bahr, Okolo, Polsgrove, & Eckert, 1988). Computational practice is usually structured hierarchically, with an emphasis on mastery of procedural steps in an algorithm as students move from easy to complex problems. Students frequently learn these rote procedures without any conceptual understanding. The logic of computational drill and practice, it seems, is that paper-and-pencil proficiency is a highly valued aspect of mathematics instruction. It is also the kind of instruction that has considerable “face validity” with many practitioners. Early versions of curriculum-based measurement (Fuchs, Fuchs, & Fernstrom, 1992, 1993), as well as the continued nature of direct instruction interventions (Carnine et al., 1997; Howell, Sidorenko, & Jurica, 1987; Kelly, Gersten, & Carnine, 1990; Stein, Silbert, & Carnine, 1997), exemplify the emphasis on computational drill and practice.

However, spending inordinate amounts of time on computational drill and practice may not be all that beneficial for students with learning disabilities. First, achieving mastery does not come easily, as many information processing studies of computational errors, or “bugs,” have suggested (Van Lehn, 1990; Woodward et al., 1999; Woodward & Howard, 1994; Woodward, Howard, & Battle, 1997). The likelihood that a student will make some kind of error is evident in a problem such as $357 \times 43$ by hand. Such tasks are best suited to readily available technological devices, such as calculators. Replacing extensive paper-and-pencil practice with the thoughtful use of calculators is certainly a key component of mathematics reform (Romberg, 1992; Usiskin, 1998), and limited research indicates that special education students benefit from calculator practice (Horton, Lovitt, & White, 1992; Woodward et al., 1999). What remains for many practitioners and researchers, however, is the question of exactly how calculators should complement or substitute for paper-and-pencil practice.

One way to shift toward a thoughtful use of calculators is to focus on the kind of conceptually guided instruction that is now advocated in the math reform literature (National Research Council, 2001) and the NCTM Standards. Over the last 15 years, conceptually guided instruction has moved from a traditional cognitive and developmental orientation (e.g., Hiebert, 1986; Skemp, 1987) to a social constructivist position (e.g., Ball, 1993; Cobb, 1999). Some special education math researchers and curriculum developers (e.g., Mercer & Mercer, 1997) have outlined an instructional agenda in which students with learning disabilities are taught a range of algorithms for basic whole number operations. The partial products algorithm, for example, makes explicit the place values in a problem like $362 \times 4$. Lampert (1986), whose work exemplifies social constructivist practice, draws on algorithms such
as partial products as a way to enhance students' conceptual understanding of multiplication.

The focus of this approach to instruction is on conceptual understanding of key topics (e.g., regrouping, place value) as well as the integrated use of calculators. Through carefully scaffolded activities using authentic tasks, students with learning disabilities could study alternative algorithms or contrast, for example, partial product methods with the traditional multiplication algorithm (see Hiebert et al., 1997). Thus, problem solving and analysis replace drill and practice, and calculators replace paper-and-pencil computation. Again, calculators can play an integral role in freeing students from the mechanics of learning how to operate on rational numbers (e.g., divide decimals).

A conceptually guided approach would also enable skill development in the area of number sense. Teachers would be in a position to focus on mental computations and estimation, aspects of number sense that follow directly from this new approach to computations. Thus, students would learn how to “round and multiply” problems like 357 x 43 into one of many forms based on the context. A context in which a liberal estimate was appropriate might allow the student to compute 400 x 40. This could be done mentally or quickly by hand, though the paper-and-pencil algorithm might be different from the traditional one that is currently taught to students. A different context requiring closer approximations may necessitate a more elaborate strategy (e.g., 350 x 40 = 300 x 40 + 50 x 40). Clearly, these skills develop over a considerable period of time; however, the knowledge gained would be far closer to what we value today in respect to competence in mathematics. At a mundane yet significant level, this knowledge is critical to assessing whether an answer on a calculator screen is correct or the result of improper keying.

This kind of an approach to computation would enable teachers to concentrate more on the topics in which whole number operations are applied. Baroody and Hume (1991) described multiple representations for teaching fractions and how they can be used with students with learning disabilities. Much of this instruction is consistent with the kind of conceptually guided teaching described in the social constructivist literature. These suggestions for reconceptualizing the way students with learning disabilities “manipulate numbers” (e.g., focus more on the conceptual side of basic operations, develop multiple strategies for approximating numbers) most certainly require further research. Although they are consistent with the Standards, they are also consistent with Hiebert’s (1999) commentary on the Standards; that is, not all practices related to the Standards have a substantive research base at this point in time. Furthermore, these suggestions also remind us of the trade-offs that are likely to occur (e.g., students with learning disabilities simply take more time to acquire mathematical knowledge). There comes a point in a student’s education (i.e., the beginning of middle school) when educators need to take into account the limited time remaining for the formal study of mathematics.

**Instruction in Problem Solving**

Problem solving is considered by some to be the centerpiece of mathematics reform (De Corte et al., 1996). Technology is increasingly replacing the need for hand calculations and accelerating the amount of data for analysis. Most important, math educators’ long-standing critique of traditional word problems has moved instruction toward complex problems (e.g., performance assessments) and sometimes highly authentic problem solving (e.g., anchored instruction).

In special education, there is limited research in problem solving, and it is often atheoretical in nature or guided largely by a behavioristic paradigm (Jitendra & Xin, 1997). Consequently, this kind of research has focused on teaching students with learning disabilities specific strategies for solving traditional word problems. These include having the students attend to key words in the problems (Darch, Carnine, & Gersten, 1984; Gleason, Carnine, & Boriero, 1990; Wilson & Sindelar, 1991), as well as providing massed or “end of the chapter” practice around one kind of strategy for solving a highly structured set of problems (Carnine et al., 1997; Moore & Carnine, 1989). Mathematics educators have soundly criticized both approaches.

Metacognitive strategy training has been an exception in this pattern of special education research (e.g., Case, Harris, & Graham, 1992; Montague, 1995, 1997; Montague & Bos, 1986). Many math reformers regard metacognition to be the central feature of problem solving (De Corte et al., 1996). Unfortunately, students with learning disabilities characteristically have a difficult time with metacognitive activities essential to effective problem solving. Their poor problem solving is also a result of difficulty in problem representation, and they tend to solve problems impulsively, with little attention to the evaluation phase. Problem representation involves having students use the domain heuristic of graphically representing word problems using relational schematics (see Jitendra, 1999). There are a number of other domain heuristics (e.g., make a simpler version of the problem and solve it, look for a pattern, work backwards) that should be taught as well, particularly in the context of a wide range of math problems, not just word problems. These are reasonable extensions of previous metacognitive research in special education. In his work on problem solving, Schoenfeld (1985) offered one model for successfully blending these domain heuristics with wider metacognitive strategies.

Recent work has focused on the role of guided instruction on complex or “anchored” problems (e.g., Bortge & Hasselbring, 1993; Goldman et al., 1997). Anchored instruction is another valuable direction in mathematics instruction because it is one of the few examples in the field that suggests how guided classroom discussions work and how they can enhance higher order thinking. It is one of the few current efforts in mathematics instruction for students with learning disabilities that draws on social constructivism in its attempt to address the “inert knowledge” problem (i.e., application of formal
knowledge in highly situated contexts; Bransford, Sherwood, Vye, & Rieser, 1986). However, the growing support for anchored instruction as situated cognition (e.g., Gersten & Baker, 1998) should be tempered by some practical and theoretical considerations. For example, authentic problems are difficult to generate, and they are not necessarily as meaningful to students as they are to adults. Critics have raised similar concerns with complex performance methods of assessment (Linn, Baker, & Dunbar, 1991). Recent work with middle school students with learning disabilities has indicated that what curriculum developers initially thought were appropriate real-world problems had little relevance to the world of 12- and 13-year-olds (Woodward, 2001). In this case, researchers modified their problems to make them more authentic to middle school students, but their research also suggested that the problems still had to be balanced by more traditional exercises. Finally, the capacity of a teacher to include students with learning disabilities adequately in whole-class discussions remains a problematic dimension of reform mathematics (for a detailed discussion of this problem, see Baxter, Woodward, & Olson, 2001).

De Corte et al. (1996) raised further concerns about highly situated instruction. They contended that highly situated problem solving does not necessarily generalize to the kinds of problem solving found in formal mathematics. For this reason, students with learning disabilities need continued opportunities to use general metacognitive and domain heuristic strategies on complex but contained problems. Under these conditions, the basic cognitive instructional principle of “less is more” applies. Instead of working through 10 to 15 one-step or “end of the chapter” word problems, students might work through one or two problems completely in a lesson, and the classroom dialogue would include discussion of strategies (i.e., general metacognitive and domain-specific heuristics), multiple solutions to problems, and a “debriefing” component that addressed what made these particular problems difficult or unique.

This shift toward an in-depth examination of one or two problems not only is likely to enhance the evaluation step in problem solving but also will enable teachers to focus much more on classroom dialogue, particularly the role of what is loosely described in the literature as scaffolding. These elements of instruction in mathematical problem solving are underdescribed in the special education literature, and on the rare occasions where they occur, they stem from the behaviorist tradition. For example, Carnine and his colleagues (Carnine, Dixon, & Silbert, 1998; Kameenui & Carnine, 1998) present scaffolding as guided practice with step-by-step feedback that is eventually faded. On other accounts, scaffolding has been relegated to external devices such as cue cards for metacognition (see Stone, 1998).

A recent discussion of scaffolding demonstrates how poorly defined this construct has become. Several prominent special educators have suggested that scaffolding has been reduced to a technique with little relation to theory (Reid, 1998; Stone, 1998; Wong, 1998). Stone (1993, 1998) also noted that research should focus on the linguistic or semiotic features of the teacher–student interaction, as well as the way scaffolding is used on an individual basis in small-group instruction. Scaffolding may be the wrong metaphor for teacher–student interactions in problem solving; a broader method, such as interactive instruction (Bos & Anders, 1990; Wong, Butler, Fitzere, & Kuperis, 1996), offers a better description of how teachers should foster problem solving in students with learning disabilities.

Regardless of how scaffolding or interactive instruction unfold in problem-solving research, there is a further need to articulate the “emotional” dimensions of problem-solving instruction. This is particularly true of late elementary and secondary students, and teachers need to be sensitive to the historical experiences of the learner. For students with learning disabilities, this often translates as the tendency to (a) make negative attributions about their capacities as problem solvers specifically and, more generally, learners; (b) place little value on any number of mathematical activities; and (c) immediately elicit support from teachers in any number of ways once the mathematical problems become challenging. Again, there is very little in the special education or at-risk literature that articulates how teachers successfully mediate these challenges and move students toward the kind of mathematical problem solving that appears in the general education reform literature. In this regard, the work of attribution theorists (e.g., Cognition, 1992; Weiner, 1992) is a helpful way of conceptualizing services for students with learning disabilities.

Conclusions

Much of the special education intervention research that is consistent with the NCTM Standards and math reform is at only an emergent stage; however, the forces sustaining reform in this country typically yield an unambiguous message. We have moved from an era of hand-computation practice to sense-making in mathematics. Although some special education researchers have investigated interventions consistent with math reform, others continue to focus on traditional topics in mathematics. What is missing is a synthesis of these approaches around topics that are commonly taught to students with learning disabilities. Even more the case, it is essential that this be done with an eye toward theory. The recent discussions around scaffolding (see Reid, 1998), for example, clearly indicate how freely constructs are reduced to the level of technique, with little regard for a guiding theoretical framework. Social constructivism, at least in its more contemporary form, provides such a framework for articulating skill development in the context of more complicated efforts.

This article provided a broad, early attempt at synthesizing where special education is with respect to mathematics reform. It is important to note that even though our discussion of mathematical topics was restricted to facts, computations,
and problem solving, we do not imply that these should be the only topics of instruction for students with learning disabilities. A host of other important topics (e.g., geometry, probability, measurement) also warrant a significant place in a student’s plan of instruction. In the end, a core concern in mathematics reform for students with learning disabilities is to go beyond the acquisition of knowledge of discrete topics, such as the ones discussed in this article, to a comprehensive, integrated understanding of the discipline. Reformers (e.g., Cohen, McLaughlin, & Talbert, 1993; Resnick, 1987) long argued that mathematics, like other disciplines, such as science or social studies, requires a particular “habit of mind” for sustained success as students move to higher and more complex levels of instruction in school. Arguably, this claim has increasing credibility as schools move to adopt reform curricula, as well as technology such as graphing calculators, as part of middle school mathematics.

We also believe that a student’s progress through these topics is likely to be uneven. For example, middle school students should be able to move relatively quickly through conceptually guided computations en route to a more competent use of calculators, based in large part on the nature of the instruction. On the other hand, facts, mental computation, and estimation will take much longer to develop, as the research on math fact acquisition cited in this article suggests. This would also be true of competence in problem solving, where variations in the overall difficulty as well as specific semantic dimensions of problems will affect student success. Current work in this area (e.g., Woodward, Monroe, & Baxter, 2001) indicates that students can be strikingly independent in solving some multistep or geometric problems after 2 months of instruction. Their success on other problems, however, indicates that they have only a partial ability to use their strategic knowledge and that a considerable level of interactive instruction is required. Successful independent use of general metacognitive and domain specific heuristics for problem solving is likely to take years to develop in students with learning disabilities.

In closing, we concur with Hiebert (1999), who suggested that articulating the implications of mathematics reform for special education should be grounded in what we value educationally. This consideration is of paramount concern for students with learning disabilities, who often have limited instructional time left before they complete their study of mathematics. Undoubtedly, there will be those in our field who will defend the “skills first” orientation to mathematics in perpetuum. However, reflection on what students with learning disabilities are likely to gain from this orientation suggests an isolated body of knowledge that has little application to the increasingly complex mathematics found at the middle and secondary school levels and even less application to a world of work filled with computing devices. It is hoped that new directions in mathematics education will help to move students with learning disabilities out of a narrow and highly procedural set of experiences closer to the kind of mathematical instruction that is valued today.

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