MCS-121
Section 5.1

Suppose oil is leaking out of a container at a decreasing rate. The rate is measured at hourly intervals and given in the following table.

Time (hours)	0	1	2	3	4
Rate (liters/hour)	35	30	26	23	21

We want to estimate the total amount of oil that has leaked out of the container during the time shown. Answering the following questions will show you how to make an estimate.

1. What is the maximum amount of oil that could have leaked out during the first hour? The minimum?

2. What is the maximum amount of oil that could have leaked out during the second hour (i.e. between $t = 1$ and $t = 2$)? The minimum? How about during the third and fourth hours?

3. During the entire four hour interval from $t = 0$ to $t = 4$, what is the maximum amount of oil that could have leaked out? The minimum? Explain where you used the assumption that the rate at which the oil was leaking was decreasing.

4. If you had to guess how much oil actually leaked in the four hour interval, what would be your guess? What is the maximum possible error in your guess? (In other words, what is the maximum possible difference between your guess and the true value?)

5. Suppose additional readings are obtained and compiled in the table:

Time (hours)	0	0.5	1	1.5	2	2.5	3	3.5	4
Rate (liters/hour)	35	33	30	27	26	24	23	22	21

Recalculate the upper and lower estimates in light of this new information. Make a new guess for the total amount of oil which has leaked out in the four hour interval and estimated the maximum possible error in your guess.

6. If readings of the rate were obtained every tenth of an hour, by how much would your upper estimate exceed your lower? What if the readings were obtained every hundredth of an hour?

7. Explain why you can calculated the total amount of oil that has leaked out to any desired degree of accuracy if you have access to readings of the rate at every instant during the four hour interval.

MCS-121
Section 5.1 - Answers

1. Maximum: 35 l
   Minimum: 30 l

2. Maximum: 30 l
   Minimum: 26 l

3. Maximum: $35 + 30 + 26 + 23 = 114$
   Minimum: $30 + 26 + 23 + 21 = 100$

4. $\frac{114+100}{2} = 107$, maximum possible error is 7

5. Max: $(35 + 33 + 30 + 27 + 26 + 24 + 23 + 22).5 = 110$ Min: $(33 + 30 + 27 + 26 + 24 + 23 + 22 + 21).5 = 103$

   Guess: 106.5, max possible error is 3.5

6. $(35 - 21).1 = 1.4$
   $(35 - 21).01 = .14$

7. The difference between the upper and lower estimates is $(35 - 21)\Delta t$. We can make that difference as small as we want by taking Δt as small as we need.

   If we calculate the average of the lower and upper estimates, the error will be no more than half the distance between the upper and lower estimates.