MCS-121 – Calculus I

Optimization

1. The sum of two non-negative numbers is 48. What is the largest possible value for the sum of their squares?

2. I want to make a fish tank with a volume of 2m^3 and I would like the base of the tank to be a rectangle twice as long as it is wide. The base and sides of the tank are to be of glass. What shape of tank will use the least amount of glass (and so cost least)?

3. A feedlot farmer has 200 feet of fencing with which to enclose two adjacent rectangular corrals of the same size. What dimensions should be used so that the enclosed area will be a maximum?

4. A farmer is planning to plant a small orchard, and is gathering information about the amount of fruit he can expect to harvest each year once the trees mature. He is advised that, if he plants up to 60 trees of a particular type on his plot of land the average harvest from each tree will be about 120 kg, but for each additional tree planted the expected yield will go down by an average of 2 kg per tree as a result of overcrowding. Naturally he wants to plant for the maximum yield of fruit. How many trees should he plant?

5. A rectangular storage container with an open top is to have a volume of 10 m^3. The length of its base is twice the width. Material for the base costs \$10 per square meter. Material for the sides costs \$6 per square meter. Find the cost of materials for the cheapest such container.

6. A power station is on one side of a river that is $\frac{1}{2}$ mile wide, and a factory is 6 miles downstream on the other side. It costs \$6 per foot to run power lines overland and \$8 per foot to run them underwater. Find the most economical path for the transmission line from the power station to the factory.

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Optimization - Answers

1. Maximize $Q = x^2 + y^2$ subject to $x + y = 48$.

   Maximize $Q = x^2 + (48 - x)^2$, $0 \le x \le 48$.

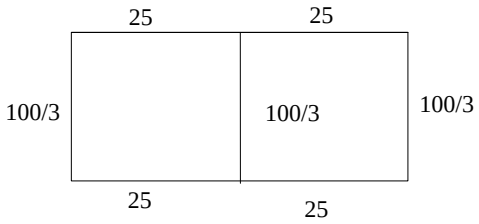
   Maximum value of Q is $48^2 = 2304$.

2. Minimize $A = lw + 2lh + 2wh$ subject to $l = 2w$ and $lwh = 2$.

   Minimize $A = 2w^2 + \frac{6}{w}$, $w \ge 0$.

   A is minimum when

   $w = 1.5^{1/3} \approx 1.14\text{m}$ $\quad l = 2(1.5^{1/3}) \approx 2.28\text{m}$ $\quad h = 1.5^{-2/3} \approx .76\text{m}$

3. 

4. Maximize $Y(t) = \begin{cases} 120t & 0 \le t \le 60 \\ (120 - 2(t - 60))t & t \ge 60 \end{cases}$

   He should plant 60 trees to maximize the yield of fruit.

5. Minimize $C = 10(lw) + 6(lh) + 6(wh)$ subject to $l = 2w$ and $lwh = 10$.

   Minimize $C = 20w^2 + \frac{360}{w}$, $w \ge 0$.

   The minimum cost is $20(9^{2/3}) + \frac{360}{9^{1/3}} \approx \$259.60$.

6. Let x be the length of line underwater and let y be the length of line overland, both in miles.

   Minimize $C = 42,240x + 31,680y = 42,240\sqrt{(6 - y)^2 + .25} + 31,680y$, $0 \le y \le 6$.

   The cost is minimized when $y = 5.43$.