Extra Optimization Problems

1. A factory makes cylindrical cans of volume 500cm^3. If they want to use the least amount of metal, what should they make the diameter and height of the can? If the metal for the top and bottom of the can costs twice as much as the metal for the sides, what should the dimensions of the can be to minimize the cost?

2. Find the coordinates of the point (x, y) on the curve $y = \sqrt{x}$ which is closest to the point $(1, 0)$?

3. Gustavus is building a new running track. It is to be the perimeter of a region obtained by putting two semicircles on the ends of a rectangle. However, due to financial constraints, the administration has decided to grow corn in the area surrounded by the track. If the track is to be 440 yards long, determine the necessary dimensions to build the track in order to maximize the area for growing corn.

4. The U.S. Postal Service will accept a box for domestic shipment only if the sum of its length and girth (distance around) does not exceed 108 in. What dimensions will give a box with a square end the largest possible volume?

5. You are designing a rectangular poster to contain 50 in^2 of printing with a 4 in. margin at the top and bottom and a 2 in. margin at each side. What overall dimensions will minimize the amount of paper used?

6. A wire 2 meters long is cut into two pieces. One piece is bent into a square for a stained glass frame while the other piece is bent into a circle for a TV antenna. To cut down on storage space, where should the wire be cut to minimize the total area of both figures? Where should the wire be cut to maximize the total area?

7. A wire 6 meters long is cut into twelve pieces. These pieces are welded together a right angles to form the frame of a box with a square base. Where should the cuts be made to maximize the volume of the box? Where should the cuts be made to maximize the total surface area for the box?

8. You have just invented a new peanut butter guacamole dip, and you open a stand in front of the student union to sell this goop by the jar. Somehow a rumor gets started, certainly not traceable back to you, that it is an aphrodisiac and sales take off. At a price of \$1.00 a jar, you sell 500 jars a day. For each nickel that you increase the price, you sell two fewer jars. Assuming that your fixed cost per day is \$200 (protection money), and the cost per jar to you is 50 cents, determine the price for which you should sell your dip in order to maximize your profit.

9. You are in a rowboat on Lake Harriet, 2 miles from a straight shoreline, taking your potential in-laws for a boat ride. Six miles down the shoreline from the nearest point on shore is an outhouse. You suddenly feel the need for its use. If you can row at 2 mph and run at 6 mph, for what point along the shoreline should you aim in order to minimize the amount of time it will take you to get to the outhouse.?

Extra Optimization Problems - Answers

1. $r = \left(\frac{1000}{8\pi}\right)^{1/3} \approx 3.41\text{cm} \qquad h = \frac{500}{\pi\left(\frac{1000}{8\pi}\right)^{2/3}} \approx 13.69$

2. $(\frac{1}{2}, \sqrt{\frac{1}{2}})$

3. $r = \frac{220}{\pi}$, $w = 0$, i.e. a circle

4. $18 \times 18 \times 36$

5. $w = 5$, $h = 10$

6. Minimize area: $8/(\pi + 2) \approx 1.56$ for square and $2 - 8/(\pi + 2) \approx .44$ for circle
   Maximize area: all wire in the circle

7. To maximize volume: 12 pieces of length 1/2, box will be a cube
   To maximize surface area: 12 pieces of length 1/2, box will be a cube

8. $\Pi(q) = -.025q^2 + 13.5q - 200 - .5q$
   $p = \$7$

9. $6 - \frac{1}{\sqrt{2}} \approx 5.23$ miles from outhouse