1. Define $f(x) = \frac{1}{(1-x)}$ and denote r iterations of the function f by f^r. Compute $f^{1999}(2000)$.

2. Suppose that there are n lines in the plane, no two parallel and no three intersecting at a point. Into how many regions is the plane divided by these lines?

3. How many ways can a $1 \times n$ rectangle be filled by nonoverlapping 1×1 and 1×2 rectangles?

4. Let $f(x) = xe^{2x}$. Let $f^{(n)}$ be the nth derivative of f. Show that $f^{(n)} = a_n e^{2x} + b_n x e^{2x}$, for some numbers a_n and b_n. Find formulas for a_n and b_n.

5. Suppose that $n \geq 3$. Show that an even number of the fractions
$$\frac{1}{n}, \frac{2}{n}, \frac{3}{n}, \ldots, \frac{n-1}{n}$$
are in lowest terms.

6. Find a formula for
$$\prod_{i=1}^{n} \left\{ 1 - \frac{4}{(2i-1)^2} \right\}.$$

7. Let $b(n)$ be the number of binary strings of length n which do now contain the substring 11. Find a formula for $b(n)$. For example, $b(3) = 5$ (the five strings are 000, 001, 010, 100, and 101).

8. (Putnam 1990) Let $T_0 = 2, T_1 = 3, T_2 = 6$, and for $n \geq 3$,
$$T_n = (n+4)T_{n-1} - 4nT_{n-2} + (4n-8)T_{n-3}.$$
The first few terms are $2, 3, 6, 14, 40, 152, 784, 5168, 40576, 363392$. Find, with proof, a formula for T_n of the form $T_n = A_n + B_n$, where $\{A_n\}$ and $\{B_n\}$ are well-known sequences.

9. A composition of a positive integer n is a summation $n = n + 1 + \cdots + n_k$, for some k, where the summands n_i are positive integers and the order of the summands matters. Let $S(n)$ be the number of compositions of n. Find a formula for $S(n)$.

10. (From Lewis Carroll.) Find three right triangles with sides of integer length all having the same area.