In-Class Problems

1. A monk climbs a mountain. He starts at 8am and reaches the summit at noon. He spends the night on the summit. The next morning, he leaves the summit at 8am and descends by the same route that he used the day before, reaching the bottom at noon. Prove that there is a time between 8am and noon at which the monk was at exactly the same spot on the mountain on both days. (Notice that we do not specify anything about the speed that the monk travels. For example, he could race at a 1000 miles per hour for the first few minutes, then sit still for hours, then travel backward, etc. Nor does the monk have to travel at the same speed going up as going down.)

2. What is the first time after 12 o'clock that the hour and minute hands meet? This is an amusing and moderately hard algebra exercise, well worth doing if you never did it before. However, the problem can be solved in a few seconds *in your head* if you avoid messy algebra and just consider the "natural" point of view.

3. A bug sits on one corner of a unit cube, and wishes to crawl to the diagonally opposite corner. If the bug could crawl through the cube, the distance would of course be $\sqrt{3}$. But the bug has to stay on the surface of the cube. What is the length of the shortest path?

4. Assume that P is a regular polygon with k sides. What is the measure of any of the k angles formed by P?

5. Can you cut an arbitrary triangle into pieces so that the pieces can be rotated and translated (but not flipped) so as to form the mirror image of the given triangle? It can be done in just two cuts.

6. Can you cut a square into seven isosceles right triangles, no two of which are congruent?

7. Triangle ABC is equilateral and P is in its interior. The distances PA, PB, PC are 3, 4, 5 respectively. What is the side-length of the triangle?

8. Suppose that T is a triangle in the plane. We let P, Q, R be the midpoints of each of the three sides. Prove that the triangle determined by P, Q, R is similar to the original triangle T.

9. Prove that any angle subtended by a semi-circle is a right angle.

10. A chord of constant length slides around in a semicircle. The midpoint of the chord and the projections of its ends upon the base form the vertices of a triangle. Prove that the triangle is isosceles and never changes its shape.

11. (Putnam 1978) Find the area of a convex octagon that is inscribed in a circle and has four consecutive sides of length 3 units and the remaining four sides of length 2 units. Give the answer in the form $r + s\sqrt{t}$, with r, s and t positive integers.