

ABSTRACT

TIGHT CLOSURE, PLUS CLOSURE AND FROBENIUS CLOSURE IN CUBICAL CONES

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Let R be a Noetherian ring of characteristic p . Given a test element c , we call R strongly bounded relative to c if there exists an R -linear map $R^{1/q} \rightarrow R^{1/pq}$ taking $c^{1/q}$ to $c^{1/pq}$ for some $q = p^e$. It is shown that if R is strongly bounded relative to a test element, then tight closure commutes with localization in R . It is also shown that if R is a one-dimensional F-finite domain then there exists a test element c such that R is strongly bounded relative to c .

Let $R = K[[x, y, z]]/(x^3 + y^3 + z^3)$, where K is a field of characteristic p and $p \equiv 2 \pmod{3}$. It is shown that for most irreducible m -primary \mathbb{Z}_3 -graded ideals $I \subseteq R$, we have $I^F = I^*$, and hence $I^* = IR^+ \cap I$. It is also shown that $I^F = I^*$ for several classes of not necessarily irreducible \mathbb{Z}_3 -graded ideals in R . It is shown that the question of whether $I^F = I^*$ in R can be reduced to the case of \mathbb{Z}_3 -graded irreducible modules.