ABSTRACT

TIGHT CLOSURE, PLUS CLOSURE AND FROBENIUS CLOSURE IN CUBICAL CONES

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Let $R$ be a Noetherian ring of characteristic $p$. Given a test element $c$, we call $R$ strongly bounded relative to $c$ if there exists an $R$-linear map $R^{1/q} \to R^{1/pq}$ taking $c^{1/q}$ to $c^{1/pq}$ for some $q = p^e$. It is shown that if $R$ is strongly bounded relative to a test element, then tight closure commutes with localization in $R$. It is also shown that if $R$ is a one-dimensional F-finite domain then there exists a test element $c$ such that $R$ is strongly bounded relative to $c$.

Let $R = K[[x, y, z]]/(x^3 + y^3 + z^3)$, where $K$ is a field of characteristic $p$ and $p \equiv 2 \mod 3$. It is shown that for most irreducible $m$-primary $\mathbb{Z}_3$-graded ideals $I \subseteq R$, we have $I^F = I^*$, and hence $I^* = IR^+ \cap I$. It is also shown that $I^F = I^*$ for several classes of not necessarily irreducible $\mathbb{Z}_3$-graded ideals in $R$. It is shown that the question of whether $I^F = I^*$ in $R$ can be reduced to the case of $\mathbb{Z}_3$-graded irreducible modules.