CZ Exercise 4.8

Student 2

Theorem 1 *If every vertex of a graph G has a degree of at least 2, then G contains a cycle.*

Proof. [Direct Proof] Suppose that G is a connected graph, with each vertex having a degree of at least 2. Pick any vertex u, since u is a part of G, a connected graph, there exists a path u,u1 such that u = u1. Since u1 has a degree of at least 2, there is another path from u1, u2. These paths create a walku, u1, u2. Let this walk continue, leaving each vertex on an edge not previously used, until u occurs that has previously occurred. This gives us a closed walk u, u1, u2, ..., u which is a cycle, C1. Therefore, G has a cycle. ∎