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Homework 2

1. Prove that the **hockeystick identity**
$$\sum_{k=0}^{r} \binom{n+k}{k} = \binom{n+r+1}{r}$$
is true for all positive integers n and r.

2. The notation $\binom{x}{k}$ can be generalized to cover all real x and all integer k as follows
$$\binom{x}{k} = \begin{cases} x^{\underline{k}}/k! & \text{if } k \text{ is a nonnegative integer} \\ 0 & \text{if } k \text{ is a negative integer.} \end{cases}$$
   (a) Determine $\Delta\binom{x}{k}$ where x is the variable and k is an integer constant.
   (b) Calculate (i) $\binom{100}{3}$, (ii) $\binom{-4}{5}$, (iii) $\binom{1.5}{3}$.

3. Apply the difference operator to each of the following functions. Simplify your answer as much as possible.
   (a) $f(x) = 3x^{\underline{3}} + 4x^{\underline{2}} - 5x^{\underline{1}} - 6$
   (b) $g(x) = x2^x$
   (c) $h(x) = \dfrac{1}{(x+3)(x+2)(x+1)}$

4. Fill in this table of forward differences $\Delta^k f(x)$ for $k = 1, 2, 3$ as much as possible.

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
0	-1			
1	2			
2	11			
3	32			
4	71			
5	134			
6	227			
7	356			
8	527			

If f is known to be a polynomial of degree at most 3, what is that polynomial?

5. Express $5x^4 - 4x^3 - 3x^2 + 2x + 1$ as a factorial polynomial, i.e., find coefficients A, B, C, D, and E such that

$$5x^4 - 4x^3 - 3x^2 + 2x + 1 = Ax^{\underline{4}} + Bx^{\underline{3}} + Cx^{\underline{2}} + Dx^{\underline{1}} + E$$

in two ways:

   (a) by using a table of Stirling numbers of the second kind, and

   (b) by Horner's method.

6. Calculate $\Delta p(x)$ where $p(x) = 5x^4 - 4x^3 - 3x^2 + 2x + 1$, the polynomial of the previous problem.

   (a) Express the answer as a factorial polynomial.

   (b) Express the answer as an ordinary polynomial.