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Homework 3

1. For each $n = 0, 1, 2, \ldots$, define $s_n := \sum_{0 \le k \le n} k^3$.
   (a) Calculate the difference table for s_n based on the first five or six values of s_n, and use Newton's forward difference formula to find a closed-form formula for s_n, assuming that it is a degree-4 polynomial.
   (b) Prove the formula correct by mathematical induction.
   (c) Find the closed-form formula for s_n again. This time by indirect Perturbation. (You should first try direct Perturbation and see that it does not work.) Does it agree with the formula you obtained earlier?
   (d) Find the closed-form formula for s_n yet again. This time by antidifferencing and using the Fundamental Theorem of Discrete Calculus. Does it agree with the formula you obtained earlier?

2. Evaluate the following definite integrals and sums. Assume that a, b are integers with $a < b$, and n is a positive integer. Compile your answers in a nice table on one page, and attach your supporting work on a separate page.

$$\int_a^b e^x\,dx \qquad \sum_{a \le k < b} 2^k$$
$$\int_a^b 2^x\,dx \qquad \sum_{a \le k < b} e^k$$
$$\int_0^n x^4\,dx \qquad \sum_{0 \le k < n} k^{\underline{4}}$$
$$\int_0^n x^{\underline{4}}\,dx \qquad \sum_{0 \le k < n} k^4$$

3. Using the Fundamental Theorem of Discrete Calculus, determine a closed-form formula for $\displaystyle\sum_{k=0}^{n} \binom{k}{m}$ valid for all integers $0 \le m \le n$. (Hint: Use the formula for differencing a binomial coefficient that you did in the last homework.)

4. Evaluate the following sums, assuming n is a positive integer.

   (a) $\displaystyle\sum_{0 \le k < n} k^2 2^k$

   (b) $\displaystyle\sum_{0 \le k < n} k^{\underline{3}} 2^k$

5. In the following pseudocode, each invocation of the procedure TASK(i,j,k) takes k steps.

```
for i ← 1 to n do
    for j ← 1 to i do
        for k ← 1 to j do
            TASK(i,j,k)
        end for
    end for
end for
```

   Determine, as a function of n, the total number of steps it takes to execute this code snippet.