San Skulrattanakulchai
Gustavus Adolphus College
April 10, 2014

Homework 4

Show the form of computation to get each answer. Calculate values of small and moderate numbers. For very large numbers, you may leave your answer in the form of an expression involving the four basic arithmetic operations $+$, $-$, $\times$, $\div$, or exponentiation.

1. A binary code is a string of 0 and 1. A ternary code is a string of 0, 1, and 2. For example, 010 is a binary code of length 3, and 120 is a ternary code of length 3. Let S_2 be the set of all binary codes of length ten. Let S_3 be the set of all ternary codes of length twelve.

   (a) How many codes in S_2 have equal number of 0 and 1?

   (b) How many codes in S_2 have more 1 than 0?

   (c) How many codes in S_3 are not binary codes?

   (d) How many codes in S_3 have equal number of 0, 1, and 2?

   (e) How many codes in S_3 do not have equal number of 0, 1, and 2?

2. (a) How many integers between 1 and 2000 are multiples of 11?

   (b) How many integers between 1 and 2000 are multiples of 11 but not multiples of 3?

   (c) How many integers between 1 and 2000 are multiples of 6 but not multiples of 4?

   (d) How many consecutive zeros occur on the right-hand end of the decimal numeral for 50!?

   (e) How many integers between 10,000 and 100,000 have only the digits 1, 3, 5, 7, 9? How many have only the digits 2, 3, 5, and 0?

3. In how many ways is it possible to seat 5 couples (i.e., 5 men and 5 women) at a round table if

(a) there are no restrictions on the seating;

(b) no two men can sit next to each other;

(c) there exist women sitting next to each other;

(d) every couple sits side-by-side;

(e) one distinguished couple sits side-by-side (the other four couples can sit anywhere)?

4. For natural numbers n, the notation $[n]$ denotes the set of all positive integers not greater than n. For instance, $[0] = \emptyset$, and $[3] = \{1, 2, 3\}$. Also, we say that a real-valued function f is *increasing* if $f(a) < f(b)$ whenever $a < b$.

   (a) How many binary relations are there from [10] to [20]?

   (b) How many functions are there from [10] to [20]?

   (c) How many increasing functions are there from [10] to [20]?

   (d) How many 1-1 functions are there from [10] to [20]?

   (e) How many onto functions are there from [10] to [10]?

5. (a) What is the number of positive integer solutions $(x_1, x_2, x_3, x_4, x_5)$ of the inequality $x_1 + x_2 + x_3 + x_4 + x_5 \leq 10$?

   (b) How many multisets of 5 or fewer elements may be formed from the multiset $\{\, 5 \cdot 1,\ 5 \cdot 2,\ 5 \cdot 3,\ 5 \cdot 4,\ 5 \cdot 5,\ 5 \cdot 6,\ 5 \cdot 7,\ 5 \cdot 8,\ 5 \cdot 9,\ 5 \cdot 10 \,\}$?

   (c) How many different orders of at least one and at most five one-scoop cones are possible at an ice cream shop that offers 20 flavors?

   (d) How many ways can you distribute 6 marshmallows to Alice, Bob, and Charlie if each child must get at least one, and assuming that marshmallows are indistinguishable?

   (e) How many different ways can one send n identical postcards to m friends if every friend must receive at least one postcard?

6. How many five-letter strings of uppercase letters are there in which there are at most four X's, at most three Y's, at most two Z's, and any valid number of the other letters?