

Homework 5

For all problems, show how you derive and calculate the numerical answers, giving exact values if you can. Otherwise, give your answer to four significant digits.

1. A pair of dice, a red and a blue, is loaded. On the red die, a 3 appears with a probability $2/7$, while each of the other outcomes appears with a probability of $1/7$. On the blue die, a 4 appears with a probability $3/8$, while each of the other outcomes appears with a probability of $1/8$. What is the probability of 7 appearing as the sum of the numbers when the two dice are rolled, assuming “independent dice”?
2. What is the conditional probability that exactly four heads appear when a fair coin is flipped five times,
 - (a) given that the first flip came up heads?
 - (b) given that the first flip came up tails?
3. Let E be the event that a randomly generated bit string of length three contains an odd number of 1s, and let F be the event that the string starts with 1. Are E and F independent?
4. Assume that the probability a child is a boy is 0.51 and that the sexes of children born into a family are independent. What is the probability that a family of five children has
 - (a) exactly three boys?
 - (b) at least one boy?
 - (c) at least one girl?
 - (d) all children of the same sex?

5. A pair of dice is rolled in a remote location and when you ask an honest observer whether at least one die came up six, this honest observer answers in the affirmative.
 - (a) What is the probability that the sum of the numbers that came up on the two dice is seven, given the information provided by the honest observer?
 - (b) Suppose that the honest observer tells us that at least one die came up five. What is the probability the sum of the numbers that came up on the dice is even, given this information?
6. Suppose that one person in 10,000 people has a rare genetic disease. There is an excellent test for the disease; 99.9% of people with the disease test positive and only 0.02% who do not have the disease test positive.
 - (a) What is the probability that someone who tests positive has the genetic disease?
 - (b) What is the probability that someone who tests negative does not have the disease?
7. Two fair dice are rolled repeatedly until the total number of spots that comes up is divisible by 3. What is the probability that this first happens on a roll number divisible by 3, i.e., roll 3 or 6 or 9 or \dots , etc.
8. A fair coin is flipped repeatedly until a heads comes up. What is the probability that this first happens on an even-numbered flip?