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Gustavus Adolphus College May 20, 2014

Homework 6

For problems on Probability Theory, show how you derive and calculate the numerical answers, giving exact values if you can. Otherwise, give your answer to four significant digits. For problems on Asymptotics, you don't have to show your work.

1. What is the expected value when a \$1 lottery ticket is bought in which the purchaser wins exactly \$10 million if the ticket contains the six winning numbers chosen from the set $\{1, 2, 3, \ldots, 50\}$ and the purchaser wins nothing otherwise?

2. Three fair, independent dice are rolled and we note the sum of the numbers that appear.

   (a) What is the expected value of this sum?

   (b) What is its standard deviation?

3. Suppose that the probability that x is in a list of n distinct integers is $2/3$ and that it is equally likely that x equals any element in the list. Find the average number of comparisons used by the linear search algorithm to find x or to determine that it is not in the list.

4. Suppose that we roll a fair die until a 6 comes up.

   (a) What is the probability that we roll the die n times?

   (b) What is the expected number of times we roll the die?

5. Rank the following functions (whose formulas are given) by order of growth; that is, find an arrangement $g_1, g_2, \ldots, g_{12}$ of the functions satisfying g_1 is $\Omega(g_2)$, g_2 is $\Omega(g_3)$, $\ldots$, g_{11} is $\Omega(g_{12})$. Partition your list into equivalence classes such that f and g are in the same class if and only if f is $\Theta(g)$.

$(\sqrt{2})^{\lg n}$	n^2	$n!$
$(\lg n)!$	$\lg^2 n$	$\lg n!$
2^{2^n}	$\ln \ln n$	$n2^n$
$(n+1)!$	$\sqrt{\lg n}$	2^n

6. For each pair of functions (A, B) (whose formulas are given) in the table below, indicate whether A is O, o, Ω, ω, or Θ of B. Assume k, ε, and c are constants such that $k \geq 1$, $\varepsilon > 0$, and $c > 1$. Your answer should be in the form Y (for "yes") or N (for "no") written in each box.

A	B	O	o	Ω	ω	Θ
$\lg^k n$	n^ε					
n^k	c^n					
$\sqrt{n}$	$n^{\sin n}$					
2^n	$2^{n/2}$					
$n^{\lg c}$	$c^{\lg n}$					
$\lg(n!)$	$\lg(n^n)$					