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Set Theory

A set is a collection of objects considered as a whole. The main concept is that of *membership*. Given an object x and a set A, we should be able to answer whether x is a member of A or not.

There are many terms whose meaning are similar to set. These include *class*, *collection*, *family*, and *space*.

A *finite set* is one whose size is a natural number. An *infinite set* has a size that cannot be given as a natural number.

Some important number sets
$\mathbb{N} = \{0, 1, 2, 3, \ldots\}$
$\mathbb{Z}$ = the set of all integers
$\mathbb{Q}$ = the set of all rational numbers
$\mathbb{R}$ = the set of all real numbers
$\mathbb{C}$ = the set of all complex numbers

Examples of sets written

- by enumeration: $\{1, 2, 3\}$
- by set former: $\{n \in \mathbb{Z} : |n| \leq 3\}$
- by formulaic set former: $\{n^2 : n \in \mathbb{N}\}$

The empty set $\emptyset$, sometimes written $\{\}$, has no member.

Two sets are equal, written $A = B$, if they contain the same elements.

Notation:

$\in$	is a member of	$\notin$	is not a member of
$\subseteq$	is a subset of	$\subset$	is a proper subset of
$\not\subseteq$	is not a subset of	$\not\subset$	is not a proper subset of
$\supseteq$	is a superset of	$\supset$	is a proper superset of
$\not\supseteq$	is not a superset of	$\not\supset$	is not a proper superset of

where

$A \subseteq B$ means "for all x, if $x \in A$ then $x \in B$."

$A \subset B$ means "$A \subseteq B$ and $A \neq B$."

$A \supseteq B$ means "$B \subseteq A$."

$A \supset B$ means "$B \subset A$."

$A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.

Set operations:

$\cup$	union
$\cap$	intersection
$-$	set difference
A^c	complement of A

where

$A \cup B$ means $\{\, x : x \in A$ or $x \in B \,\}$.

$A \cap B$ means $\{\, x : x \in A$ and $x \in B \,\}$.

$A - B$ means $\{\, x : x \in A$ and $x \notin B \,\}$.

A^c means $U - A$ (where U is the "universal set").

Sets A and B are *disjoint* if their intersection is empty, i.e., $A \cap B = \emptyset$.

Venn-Euler Diagram can help one understand why some theorem is true, or even suggest a proof.

Some rules governing set operations:

$A \cup B = B \cup A$	(commutativity of union)
$A \cap B = B \cap A$	(commutativity of intersection)
$(A \cup B) \cup C = A \cup (B \cup C)$	(associativity of union)
$(A \cap B) \cap C = A \cap (B \cap C)$	(associativity of intersection)
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	(distributivity)
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	(distributivity)
$(A^c)^c = A$	(double complement)
$(A \cup B)^c = A^c \cap B^c \qquad (A \cap B)^c = A^c \cup B^c$	(DeMorgan's laws)

The power set $\mathcal{P}(A)$ (or 2^A) of A is the collection of all subsets of A. In other words, $\mathcal{P}(A) = \{S : S \subseteq A\}$.

Theorem. *If A is a finite set, then $|\mathcal{P}(A)| = 2^{|A|}$.*

Proof. . . . □

For $a, b \in \mathbb{Z}$ with $a \leq b$, we write $\{a, \ldots, b\}$ to denote $\{i \in \mathbb{Z} : a \leq i \leq b\}$. For $n \in \mathbb{N}$ we write $[n]$ for $\{1, \ldots, n\}$.

Let $a, b \in \mathbb{R}$ with $a \leq b$. The notation $[a, b]$ denotes the **closed interval**

$$\{x \in \mathbb{R} : a \leq x \leq b\}.$$

The notation (a, b) denotes the **open interval**

$$\{x \in \mathbb{R} : a < x < b\}.$$