MCS-265: The Theory of Computation Handout #T1
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Problems

Example Problems in $\mathcal{P}$.

PATH
INSTANCE: $\langle D, s, t\rangle$ where D is a digraph, s and t are distinct vertices of D.
QUESTION: Does D contain any directed s,t-dipath?

SPATH
INSTANCE: $\langle D, s, t, k\rangle$ where D is a digraph, s and t are distinct vertices, and k is a nonnegative integer.
QUESTION: Does D contain a dipath of length at most k from s to t?

RELPRIME
INSTANCE: $\langle x, y\rangle$ where x and y are positive integers.
QUESTION: Are x and y relatively prime?

Example Problems in $\mathcal{NP}$.

ANY-HAMPATH
INSTANCE: $\langle D\rangle$ where D is a digraph.
QUESTION: Does D contain a dipath that covers every vertex exactly once?

ANY-UHAMPATH
INSTANCE: $\langle G\rangle$ where G is a graph.
QUESTION: Does G contain a path that covers every vertex exactly once?

CLIQUE
INSTANCE: $\langle G, k\rangle$ where G is a graph and k is a positive integer.
QUESTION: Does G contain a subset V' of vertices of size $\geq k$ such that any two vertices in V' are adjacent?

DOMINATING-SET
INSTANCE: $\langle G, k\rangle$ where G is a graph and k is a positive integer.
QUESTION: Does G contain a subset V' of vertices of size $\leq k$ such that every vertex of G is either in V' or is adjacent to some vertex in V'?

DOUBLE-SAT
INSTANCE: $\langle \varphi\rangle$ where φ is a boolean formula in CNF.
QUESTION: Does φ have at least two satisfying assignments?

FEEDBACK-VERTEX-SET
INSTANCE: $\langle D, k\rangle$ where D is a digraph and k is a positive integer.
QUESTION: Is there a subset V' of vertices of size $\leq k$ such that every dicycle in D passes through at least a vertex in V'?

HAMPATH
INSTANCE: $\langle D, s, t\rangle$ where D is a digraph, s and t are distinct vertices.
QUESTION: Does D contain an s,t-dipath that covers every vertex exactly once?

HAMCYCLE
INSTANCE: $\langle D\rangle$ where D is a digraph.
QUESTION: Does D contain a dicycle that covers every vertex exactly once?

INDEPENDENT-SET
INSTANCE: $\langle G, k\rangle$ where G is a graph and k is a positive integer.
QUESTION: Does G contain a subset V' of vertices of size $\geq k$ such that no two vertices in V' are adjacent?

LPATH
INSTANCE: $\langle D, s, t, k\rangle$ where D is a directed graph, s and t are distinct vertices, and k is a positive integer.
QUESTION: Does D contain a simple s,t-path of length $\geq k$?

SAT
INSTANCE: $\langle \varphi\rangle$ where φ is a boolean formula.
QUESTION: Is φ satisfiable?

SUBGRAPH-ISOMORPHISM
INSTANCE: $\langle G_1, G_2 \rangle$ where G_1 and G_2 are undirected graphs.
QUESTION: Does G_1 contain a subgraph isomorphic to G_2?

SUBSET-SUM
INSTANCE: $\langle S, t \rangle$ where S is a collection (multiset) of positive integers, and t an integer.
QUESTION: Does S contain a subcollection T that sums to t?

TRAVELING-SALESPERSON-PROBLEM (TSP, undirected version)
INSTANCE: $\langle G, w, k \rangle$ where $G = (V, E)$ is a complete graph every edge e of which has an integral weight $w(e)$ associated with it, and k is an integer.
QUESTION: Does G contain a cycle covering all vertices whose sum of the edge weights in this cycle is $\leq k$?

UHAMCYCLE
INSTANCE: $\langle G \rangle$ where G is a graph.
QUESTION: Does G contain a cycle that covers every vertex exactly once?

UHAMPATH
INSTANCE: $\langle G, s, t \rangle$ where G is a graph, s and t are vertices.
QUESTION: Does G contain an s, t-path that covers every vertex exactly once?

VERTEX-COVER
INSTANCE: $\langle G, k \rangle$ where G is a graph and k is a positive integer.
QUESTION: Does G contain a subset V' of vertices of size $\leq k$ such that every edge in G is incident to some vertex in V'?