

# UVa 11151 - Longest Palindrome

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# Longest Palindrome

## Definition

A palindrome is a string that reads the same from the left as it does from the right. For example, I, GAG and MADAM are palindromes, but ADAM is not. Here, we consider also the empty string as a palindrome.

From any non-palindromic string, you can always take away some letters, and get a palindromic subsequence. For example, given the string ADAM, you remove the letter M and get a palindrome ADA.

## Problem

Write a program to determine the length of a longest palindrome you can get from a string.

## Solution by Dynamic Programming

Assume input string  $x = x_1x_2 \dots x_n$  is given.

For  $1 \leq i \leq j \leq n$ , define  $p(i, j)$  to be the length of a longest palindromic subsequence of the substring  $x_ix_{i+1} \dots x_{j-1}x_j$ .

# Recurrence

$$p(i, j) = \begin{cases} 0 & \text{if } i > j \\ 1 & \text{if } i = j \\ \max\{p(i+1, j), p(i, j-1)\} & \text{if } i < j, x_i \neq x_j \\ 2 + p(i+1, j-1) & \text{if } i < j, x_i = x_j. \end{cases}$$

The *artificial base case* of  $i > j$  is for convenience. (See the last recursive case.)

We seek  $p(1, n)$ .

When filling in the table, the recurrence forces the index  $i$  to be decreasing and index  $j$  to be increasing.

# Running Time

This is a 2D dynamic programming problem with a running time of  $O(n^2)$ .