

Introduction to MCS 220 and L^AT_EX

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1 Introduction to MCS 220

MCS 220, *Introduction to Analysis*, carries a **WRITD** (writing in the discipline) designation. What this means to you is that not only will you be learning mathematics, but you will also be learning the rhetoric of mathematics as well. You may think that doing mathematics means doing lots of integrals, derivatives, and other fancy calculations and while this is certainly a big part of what we do it is far from the most important part. Mathematics is primarily about theorems and their proofs. The important thing to understand right now is that a theorem is a precisely stated logical statement and a proof provides the deductive reasoning that demonstrates the truth of the theorem.

This course provides an introduction to the mathematical foundations of calculus and hence a good portion of modern mathematics. Ideas that you have taken for granted (such as continuity for example) will be discussed in depth and in a very rigorous fashion.

We will also spend a lot of time working on the writing of mathematics. Be warned, writing mathematical theorems and proofs is very different than writing a research paper. There is no place for emotional arguments or flowery exposition. Everything must be logical deductive reasoning. We start at point A (the hypothesis of the theorem) and deductive reasoning leads us through a sequence of steps until we reach point B (the conclusion of the theorem).

2 A Very Gentle Introduction to L^AT_EX

Because this is a writing course, you will be typing almost all of your assignments and you will be doing it in a mathematical typesetting language known as L^AT_EX. Don't get me wrong, I like *Microsoft Word* as much as the next guy. But it is nearly impossible to typeset mathematics in it. Thus I think that it is definitely worth the time and effort to learn L^AT_EX which was designed for this task and this task only.

2.1 Getting L^AT_EX

We have L^AT_EX on all of the computers in the lab so you can always work there. If you want, you can also download it (it's free!) onto your Mac or PC. To download L^AT_EX go to <http://www.latex-project.org/ftp.html>. You will also need an editor (something to write write your L^AT_EX in). The Mac version comes with an editor. I'm not sure if the Windows version does or does not. I like an editor called *emacs* and it is what I will be using in class. It is also available free (for Macs go to <http://aquamacs.org/> for the version I use. For Windows go to <http://www.xemacs.org/Download/index.html>. If you have any questions about installing any of this software see Aaron Nienow who manages the MCS Department computer resources or me (as a last resort). This was all very easy on my Mac so hopefully it will be easy on your computer as well.

2.2 Using L^AT_EX

L^AT_EX is much different than *Word* in that it is **not** WYSIWYG (What You See Is What You Get). You first write a L^AT_EX file like this one and then you compile it to get the “pretty” output file (ultimately a PDF file). This probably seems like a ton of work, but it turns out that it is usually faster, easier, and better looking than anything you could ever do in **Word**.

Every L^AT_EX document begins with a `\documentclass` statement that tells the compiler the type of document being produced and some default settings. You can always use the one at the beginning of this document. The meat of the document begins with a `\begin{document}` statement and ends with a `\end{document}` statement. Everything that you want to say needs to be between these two lines.

In \LaTeX regular text is generally typed normally. You don't have to worry about word spacing or paragraph spacing or line spacing etc. Every \LaTeX command begins with a backslash (\backslash).

A new paragraph is preceded with a blank line. In-line mathematics is enclosed in dollar signs such as $x_1=4$ or $f'(x)=3x^2-2$ which results in $x_1 = 4$ or $f'(x) = 3x^2 - 2$. Note that subscripts are indicated using the underscore ($_$) while superscripts with the caret ($^$). Subscripts or superscripts of more than one character must be included in curly braces (M_{12}) gives M_{12}).

Multi-line output is often bracketed by $\begin{whatever}$ and $\end{whatever}$ statements. For example displayed equations are bracketed by $\begin{equation}$ and $\end{equation}$ statements. Displayed equations can also be numbered such as the Fourier series below.

$$\sum_{n=0}^{\infty} B_n \sin\left(\frac{n\pi x}{L}\right) \tag{1}$$

Equations that are not numbered (but still displayed) are bracketed by the shortcut $\left[$ and $\right]$.

$$B_n = \frac{2}{L} \int_0^L f(x) \sin\left(\frac{n\pi x}{L}\right) dx$$

There are a couple of other things to note in these examples.

- Fractions are displayed using the $\frac{\{numerator\}}{\{denominator\}}$ command. The numerator goes in the first set of braces and the denominator in the second.
- The $\left($ and $\right)$ command resize parentheses to match the size of what's inside. You can use \left and \right with any grouping symbols.
- The $\label{eq:fourier1}$ line provides a label for referencing purposes. A numbered equation can be referred to by using its label as in the Fourier series is given in equation 1. This way you don't have to keep track of equation numbers manually. Note here how I refer to the equation. The \sim keeps the equation number "connected" to the word equation for line breaking purposes. I try to give descriptive labels to my equations.

Sometimes you may want to display several equations together like below

$$\begin{aligned} 4 &= \epsilon \pm 2\delta \\ 58 &= 8x \cdot 2y + \dots \end{aligned}$$

Some notes on this list of equations

- A displayed list of equations is created using the `\begin{eqnarray*}` `\end{eqnarray*}` delimiters.
- The asterick suppresses equation numbering.
- What you want aligned (usually the equal sign) is bracketed by ampersands (`&`).
- A new line is indicated by `\\`.
- I used a bunch of math symbols in this example. In particular symbols for greek letters are simply their name (i.e. `\epsilon`). For a capital greek letter capitalize the first letter (i.e. `\Delta` gives Δ). There is a quick reference to these symbols in the \LaTeX manual or on line at <http://stdout.org/winston/latex/>.

That's a pretty good introduction to basic mathematical typesetting using \LaTeX . Now let's talk about theorems and such. The statement of a theorem is bracketed within the `\begin{theorem}`, `\end{theorem}` statements. Similar statements are available for definitions, corollaries, lemmas, etc. See the preamble of this document for a list of these environments. The proof begins with the `\pf` command and ends with a `\qed` (again see the preamble). Here is an example.

Theorem 1 *Assume $a, b, c,$ and d are not all zero. If $ad - bc \neq 0$ then system of equations*

$$ax + by = 0 \tag{2}$$

$$cx + dy = 0 \tag{3}$$

has a unique solution of $x = 0, y = 0$.

Proof: Since $ad - bc \neq 0$ at least one of the four constants is non-zero. Without loss of generality assume $a \neq 0$. Solving equation 2 for x implies

$$x = -\frac{b}{a}y. \tag{4}$$

Substituting this into equation 3 and simplifying gives

$$\frac{ad - bc}{a}y = 0. \tag{5}$$

Since $ad - bc \neq 0$ by assumption, equation 5 implies that $y = 0$. Equation 4 then implies that $x = 0$. Hence the system of equations has $x = 0, y = 0$ as a unique solution. \square