

## PHY 230 Midterm 1 Review

Below is a summary of the things you should know for our second midterm exam. The exam is on **Friday, March 26 in class**. Below are the exam rules as they appear on the test.

### Rules:

1. You may use 1  $8.5 \times 11$  inch note sheet for use during the test. *This is the only reference you are allowed to use.*
2. You have *one hour* to complete the exam.
3. Unless otherwise noted, calculators may be used to evaluate integrals.
4. No discussion of the exam is allowed.

The exam will cover a variety of topics in linear algebra, differential equations, and Fourier series. Topics and the relevant sections are summarized below.

**Systems of Linear Equations and Row Reduction (3.2)** The basic techniques for solving systems of linear equations. Row reduction is a method for solving systems of equations by reducing them to a simpler system. Allowable operations are interchanging rows, multiplying any row by a constant, and multiplying a row by a constant and adding it to another row. There are 3 possible outcomes. There can be a unique solution. There can be no solution. There can be infinitely many solutions.

**Determinants (3.3)** Methods for computing the determinant of a matrix. Some helpful facts that can be used to simplify some calculations are in the shaded boxes on pages 88 and 89. Cramer's rule is a method for solving systems of linear equations using determinants.

**Vectors (3.4)** Review of basic vector facts. Of special note is the dot product and cross product.

**Matrix Operations (3.6)** Addition, subtraction, multiplication and other operations with matrices. Be careful with matrix multiplication as it is **not** commutative, i.e.  $AB \neq BA$  except in very special cases. Recall that two matrices can be multiplied only if the number of columns of the first matrix equals the number of rows of the second matrix. In other words, if  $A$  is  $n \times m$  and  $B$  is  $m \times k$  then  $AB$  makes sense but not  $BA$ . The size of  $AB$  is  $n \times k$ . Also important in this section is the inverse of a matrix (only for square matrices.) Not every matrix is invertible, but if  $A$  is invertible then  $A^{-1}$  is the unique matrix such that  $AA^{-1} = A^{-1}A = I$  where  $I$  is the identity matrix. Matrix inverses can be used to solve  $Ax = b$  for  $x$  by computing  $A^{-1}b$ .

**Linear Combinations etc. (3.7)** A basic overview of linear combinations and linear functions.

**Linear Dependence and Rank (3.8)** An overview of the idea of linear dependence and independence. A set of vectors  $\{v_1, \dots, v_k\}$  is linear dependent if there exists a set of constants  $\{a_1, \dots, a_k\}$  not all zero such that  $a_1v_1 + \dots + a_kv_k = 0$ . The *rank* of a matrix is the number of non-zero rows remaining after performing row reduction.

**Eigenvalues and Eigenvectors (3.11)** Eigenvalues and eigenvectors are only associated with square matrices. The eigenvectors are non-zero vectors such that  $Av = \lambda v$ . The scalar  $\lambda$  is known as the eigenvalue of the eigenvector  $v$ . To find eigenvalues we solve  $\det(A - \lambda I) = 0$ . If  $A$  is  $n \times n$  then this is a degree  $n$  polynomial and has  $n$  roots counting multiplicity. For each eigenvalue you find the corresponding eigenvector by solving  $(A - \lambda I)v = 0$  for  $v$  and only accepting non-zero solutions. Recall that this looks like  $n$  equations and  $n$  unknowns but there are really fewer than  $n$  independent equations. Thus there will always be some choice involved. Thus eigenvectors are unique only up to scalar multiplication. Eigenvalues can be real or complex. If the matrix is real and has complex eigenvalues then they come in complex conjugate pairs  $a \pm ib$ . Similarly, eigenvectors come in complex conjugate pairs. If  $v_1 = v_r + iv_i$  is the eigenvector of  $a + ib$  then  $v_2 = v_r - iv_i$  is the eigenvector of  $a - ib$ .

**Orthogonality (3.14)** The idea of an inner product  $\langle f, g \rangle$  and how it generalizes the dot product you are already familiar with. The most important fact is that given an inner product then two non-zero vectors  $f$  and  $g$  are orthogonal if and only if  $\langle f, g \rangle = 0$ . This will be the fundamental idea underlying the development of Fourier series and similar tools.

**General Vector Spaces (3.14)** You should know the definition of a vector space. In particular, you should be able to determine whether a given set of “vectors” and operations of “addition” and “scalar multiplication” define a vector space or not.

**DE basics (8.1)** You should understand that a first-order differential equation defines a slope field and that solutions to the differential equation “follow the slopes.” You should understand what it means for a function to be a solution to a given differential equation.

**Separable equations (8.2)** These are first order differential equations. The variables are separated (i.e.  $y$ 's on one-side and  $t$ 's on the other), both sides are integrated, and then one solves for  $y$ . An initial conditions is needed to determine the constant.

**Integrating factors (8.3)** You should know how to use the integrating factor method to solve differential equations of the form  $y' + p(t)y = q(t)$ .

**$ay'' + by' + cy = 0$  (8.5)** Compute the roots of the characteristic polynomial  $ar^2 + br + c = 0$  and these give you the solution. Initial conditions are needed to determine the constants. You should be able to find the real solution if the roots are complex conjugates. This is also where we talked about the complex exponential  $e^{i\theta}$  and you should be able to use this as well.

**$ay'' + by' + cy = \mathbf{F}(t)$  (8.6)** The solution is  $y(t) = y_h(t) + y_p(t)$  where  $y_h$  is the solution to the corresponding homogeneous equation (right hand side equal to 0) and  $y_p(t)$  is a particular

solution.  $y_p$  is found by the method of undetermined coefficients (i.e. guess what you see on the right hand side but with arbitrary coefficients to be determined.)

**Basic facts of periodic functions (7.2)** We didn't talk much about this in class but I did have you read this section. It covers amplitude, period, frequency. etc. In particular, you should be able to use Euler's formula ( $e^{i\theta} = \cos \theta + i \sin \theta$ ) and manipulate complex numbers.

**Real Fourier coefficients (7.5)** How to compute real Fourier series of periodic function of period  $2\pi$ .

**Fourier series on other intervals (7.8)** Adjustments to the Fourier series and coefficient formulas for functions of period  $2\ell$ .

**Even and odd functions (7.9)** Simplifications to Fourier series and coefficient calculations if the function is either even or odd.