

PHY 230 Midterm 2 Review

Below is a summary of the things you should know for our second midterm exam. The date of the exam is Wednesday, April 23 and will be taken in class. Below are the exam rules as they appear on the test.

Rules:

1. You may use 1 8.5×11 inch note sheet.
2. Unless otherwise noted, calculators may be used to evaluate integrals.
3. No discussion of the exam is allowed.

The exam will cover series solutions of differential equations and related topics. Of particular importance is orthogonality and its uses. Topics and the relevant sections are summarized below.

Basic facts of periodic functions (7.2) We didn't talk about this in class but I did have you read this section. It covers amplitude, period, frequency. etc.

Real Fourier coefficients (7.5) How to compute real Fourier series of periodic function of period 2π .

Dirichlet conditions (7.6) These tell us about convergence. In particular, if the function periodic function $P(x)$ is continuous at x_0 then the Fourier series converges to $P(x_0)$ and if the function is **not** continuous at x_0 then the series converges to the average value of the left and right hand limits.

Complex Fourier series (7.7) Fourier series of period 2π functions using the complex exponential functions e^{ikx} . Recall that Euler's formula is $e^{ix} = \cos x + i \sin x$. and that can be used to compute values of $e^{i\theta}$ for any real number θ . In particular, $e^{2\pi i} = 1$, $e^{\pi i/2} = i$, $e^{\pi i} = -1$, $e^{3\pi i/2} = -i$ etc.

Fourier series on other intervals (7.8) Adjustments to the Fourier series and coefficient formulas for functions of period 2ℓ .

Even and odd functions (7.9) Simplifications to Fourier series and coefficient calculations if the function is either even or odd.

The Series Method (12.1) The basic method for solving second-order differential equations with non-constant coefficients. Assume $y = \sum_{k=0}^{\infty} a_k x^k$, differentiate and substitute into the given DE. Your goal is to collect terms into one series with each term having the same power of x . The tools for doing this are (1) reindexing and (2) evaluating the first few terms of a series to make the counters start together. This then gives recursion relationships that allow you to compute the coefficients a_k .

Legendre's Equation: $(1 - x^2)y'' - 2xy' + \ell(\ell + 1)y = 0$ (12.2) The series method was used to solve this differential equation. We saw that if ℓ is a positive integer then there is a polynomial solution of degree ℓ . These are known as *Legendre polynomials* and are denoted by $P_\ell(x)$. Moreover, if ℓ is even the P_ℓ is an even function. Similarly if ℓ is odd.

Rodrigues' Formula (12.4) A derivative formula for computing the Legendre polynomials. This uses the ideas of 12.3.

Generating Function for Legendre Polynomials A function

$$\Phi(x, h) = (1 - 2xh + h^2)^{-1/2} = \sum_{\ell=0}^{\infty} P_\ell(x)h^\ell.$$

In other words, the coefficients of this power series are the Legendre polynomials. This is useful for establishing relationships among the Legendre polynomials. It also establishes the connection between these functions and solutions to problems in electrostatic or gravitational potential problems.

Complete sets of Orthogonal Functions (12.6) The idea of an inner product $\langle f, g \rangle$ and how it generalizes the dot product you are already familiar with. The most important fact is that given an inner product then two non-zero functions f and g are orthogonal if and only if $\langle f, g \rangle = 0$. This was the fundamental idea underlying the development of Fourier series as well as Legendre series.

Orthogonality of the Legendre Polynomials (12.7 and 12.8) We showed that if

$$\langle f, g \rangle = \int_{-1}^1 f(x)g(x)dx$$

then the Legendre polynomials are orthogonal with respect to this inner product. We also showed that

$$\langle P_\ell, P_\ell \rangle = \int_{-1}^1 P_\ell^2 dx = \frac{2}{2\ell + 1}.$$

Legendre Series How to compute Legendre series of a function defined on the interval $[-1, 1]$.

Associated Legendre Functions (12.10) Will not appear on the exam!

Generalized Power series method (12.11) This method generalized the series method described in (12.1) and is used for differential equations of the form $a(x)y'' + b(x)y' + c(x)y = 0$ where $a(0) = 0$. You assume that $y = x^r \sum_{k=0}^{\infty} a_k x^k$, differentiate and substitute into the given DE. When you differentiate **do not** adjust the starting value of the series. Your goal is to collect terms into one series with each term having the same power of x . The tools for doing this are (1) reindexing and (2) evaluating the first few terms of a series to make the counters start together. Note that when reindexing **do not** make the r go away i.e if you have x^{m+r-1} then reindex by letting $k = m - 1$. After doing this you then determine what values of r are allowable. You then use these and the recursions to compute the coefficients a_k .

Bessel's Equation: $x^2y'' + xy' + (x^2 - p^2)y = 0$ (12.12) An example of the generalized series method. One set of solutions are *Bessel functions of the first kind* denoted $J_p(x)$. We briefly talked about Bessel functions when p was not an integer. These used the gamma function $\Gamma(x)$ in their definition. **No questions about the gamma function will be asked.**

Graphs and zeros of Bessel Functions (12.14) We saw that these functions look “like” a damped spring. In particular, each Bessel function has infinitely many x -intercepts or zeros which are not generally evenly spaced. Of special importance is that the Bessel Functions are bounded (i.e. no vertical asymptotes.)

Recursion Relations (12.15) There are many relationships between the Bessel Functions and to trigonometric functions. We discussed several of these. You will not be responsible for knowing these. If you need them on the exam I will provide you with them.

Here are a couple of other notes.

1. On the exam I have consistently used A_k to denote cosine coefficients, B_k to denote sine coefficients, and C_k to denote complex exponential coefficients. This is consistent with lectures and your text. I have also used words to describe what I mean so there should be no confusion.
2. One question on the exam is a True/False question with justification. A correct answer without justification is worth only 1 point. Justifications will be graded on a partial credit basis.

That about does it. If anyone notices anything missing or confusing, let me know via email and I will correct it, clarify it, add it, or whatever.

good luck.