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3/12/2009

The Physics of a Senseless Bungee Jumper

We begin with the description of some foolish bungee jumper, determined to place his safety solely in the hands of our abilities with differential equations. We know that this bungee jumper has a mass *m*=72 kg and a drag coefficient of *a*=2.8. This being a terrestrial bungee jumper at around sea level, we take the acceleration of gravity, g, to be 9.8 m/s.

Equipped with various bungee cords of length 30 m (including a climbing rope of the same length), we begin by describing the jumper's motion as he is in free fall with only air resistance to retard his speed (the bungee cord does not yet have a restoring force because it has not yet reached it's equilibrium position).

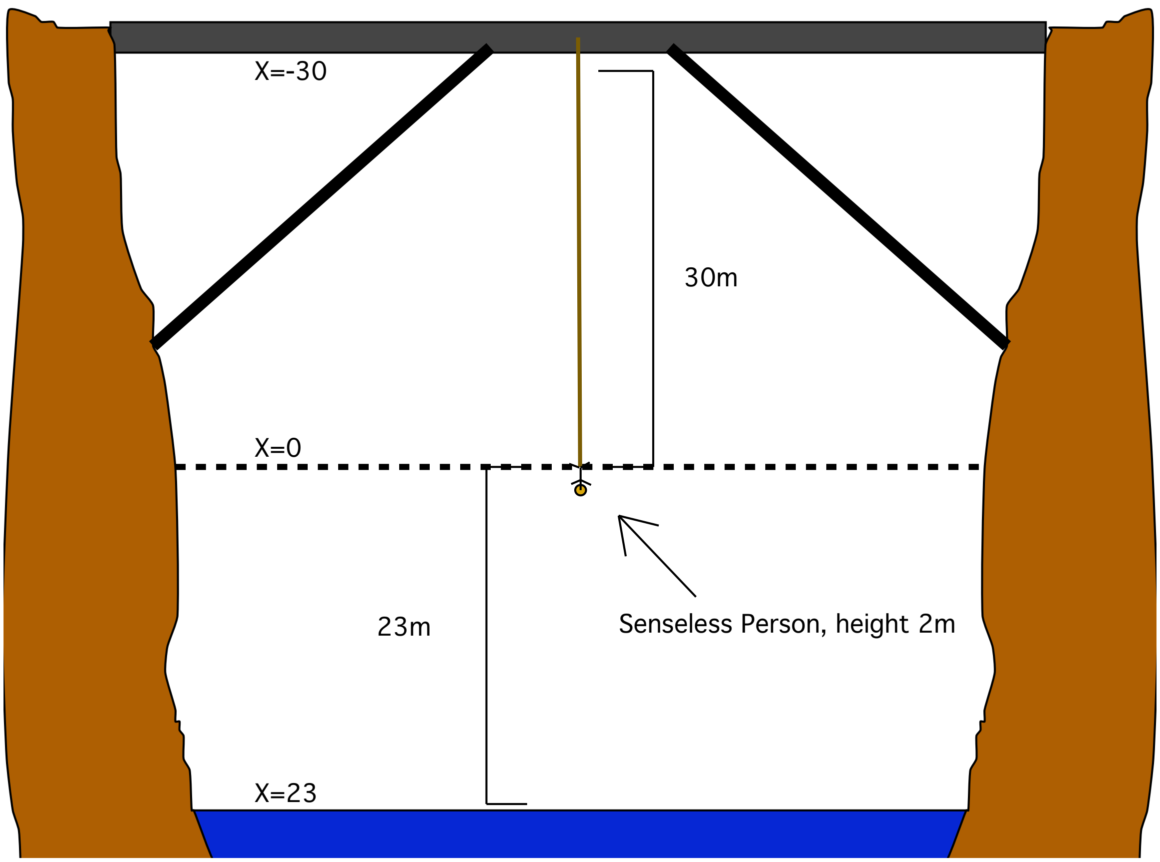

Figure 1: Picture of the Situation

The bungee jumper's motion can be described with the following differential equation, where x_f, denotes the jumper's position at time *t*:

$$mx_f'' + ax_f' = mg$$

To solve this differential equation we first found the homogeneous solution:

$$x_h(t) = C_1 + C_2 e^{-\frac{7}{180}t} + \frac{3528}{5} C_3 t$$

Next, we guessed what we saw to find the particular solution:

$$x_p(t) = \frac{mgt}{a} = 252t$$

Given the initial conditions shown in **Figure 1**,

$$x_f(0) = -30$$

$$x_f'(0) = 0$$

And finding the derivative of the general solution to be:

$$x_f'(t) = -\frac{7}{180} C_2 e^{-\frac{7}{180}t} + 252$$

We can solve for the constants C_1 and C_2 to find

$$x_f(t) = 6480 e^{-\frac{7}{180}t} + 252t - 6510$$

$$x_f'(t) = -252 e^{-\frac{7}{180}t} + 252$$

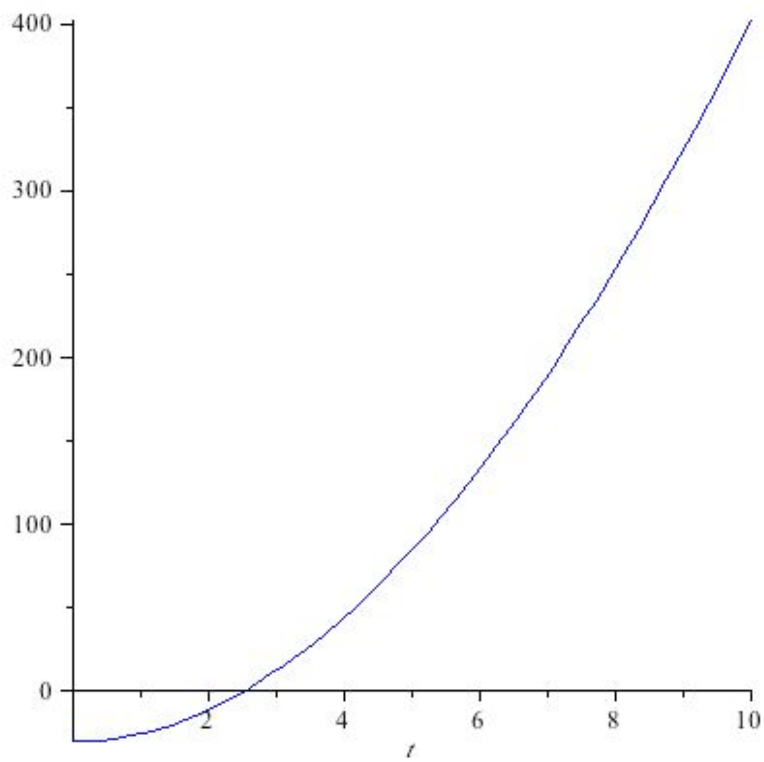

Figure 2*: $x_f(t)$*

Setting *x* equal to *0* and solving for *t* gives the time the bungee jumper takes to fall the first 30 meters.

$$t_1 = 2.51 \text{ s}$$

This value seems reasonable given that with no air resistance the time taken to fall 30 meters is 2.47 s.

Plugging t_1 into $x_f'(t)$ gives the downward speed of the bungee jumper as the bungee cord begins to stretch.

$$v_1 = x_f'(t_1) = 23.43 \text{ m/s}$$

Once the bungee cord has reached its equilibrium position we have to use a new differential equation that takes into account the bungee cord's restoring force. The new differential equation is expressed as:

$$mx'' + ax' + kx = mg$$

The homogeneous solution to the equation has the form

$$x_h(t) = e^{Lt}(A\cos(wt) + B\sin(wt))$$

Where

$$L = \frac{-a}{2m}$$

$$w = \frac{\sqrt{|a^2 - 4mk|}}{2m}$$

The particular solution is found to be

$$x_p(t) = C = \frac{mg}{k}$$

Yielding the general solution

$$x_s(t) = e^{Lt}(A\cos(wt) + B\sin(wt)) + C$$

The derivative of the general solution is found to be:

$$x_s'(t) = A(e^{Lt}\cos(wt) - we^{Lt}\sin(wt)) + B(Le^{Lt}\sin(wt) + we^{Lt}\cos(wt))$$

Setting $x(t)$ equal *to 0* and $x'(t)$ equal to v_1 and taking t to be t_1, A and B can be isolated via linear algebra (See appendix for details).

$$A = \frac{-\sin(wt_1)(v_1 + LC) - Cw\cos(wt_1)}{we^{Lt_1}}$$

$$B = \frac{\cos(wt_1)(v_1 + LC) - Cw\sin(wt_1)}{we^{Lt_1}}$$

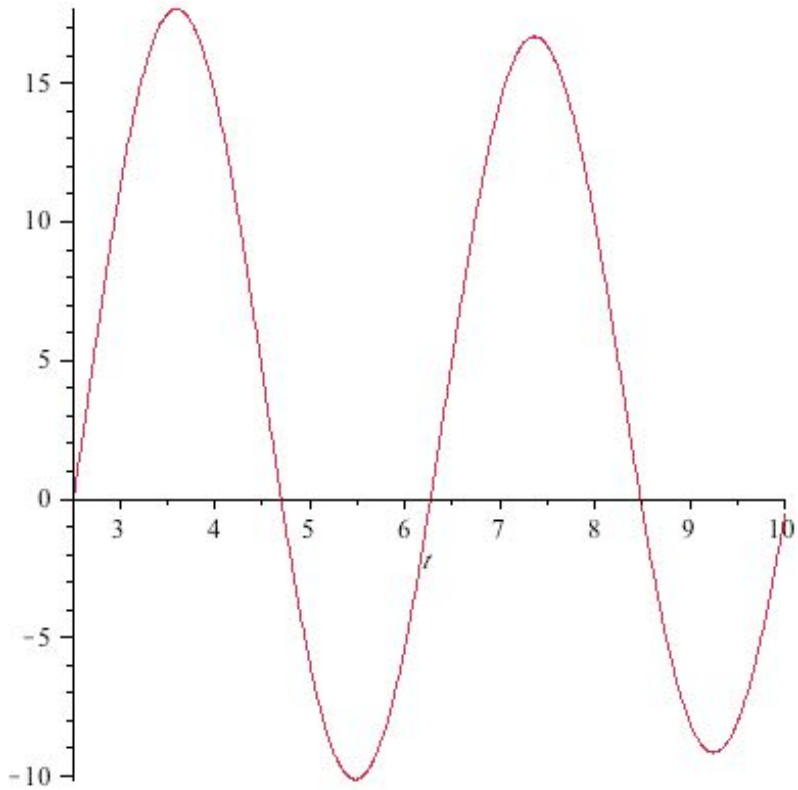

Figure 3: Graph of $x_s(t)$ with $k=200$

Since the values of A, B and C are functions of k, we can create a table (**Table 1**) of each of their values for each cord brought a long including the example cord where $k=200$. Plugging these values into $x'_s(t)=0$ and solving for t yields t_2, the time at which the bungee jumper is the closest to the water. Plugging this t_2 into x_s yields the position of the bungee jumper below the bungee cord's equilibrium point. Adding 32m (since the jumper is 2 meters tall) to x_2 yields the total distance he will fall. If x_t is greater than 53m, he gets wet.

k	A	B	C	t_2	x_2	x_t
200	14.57	-4.26	3.53	3.59s	17.44m	49.44m
102	4.01	-21.43	6.92	4.10s	26.50m	58.50m
196	14.55	-4.85	3.60	3.60s	17.70m	49.70m
284	11.12	5.94	2.48	3.40s	14.13m	46.10m

Table 1

As can be seen, the bungee cord where $k=102$ is certainly not ideal, should the bungee jumper choose to use it he will find himself very wet, and perhaps dead. The climbing rope where $k=284$, while effective in not submerging the bungee jumper, is likely a painful choice and is more likely to result in broken legs and a YouTube video rather than an enjoyable experience. The ideal choice then seems to be the bungee cord where $k=196$. This cord does not submerge the jumper in the river while at the same time delivers what is likely to be a relatively safe experience – at least as far as bungee jumping goes.