

PHY 230 Midterm 2 Takehome Problem¹.

Rules:

1. You may use your books and your notes.
2. Unless otherwise noted, computers or calculators may be used to evaluate integrals.
3. Discussing this exam with members of other groups is not allowed.

Honor Pledge: On my honor, I pledge that I have not given, received, or tolerated others' use of unauthorized aid in completing this work.

Signature 1: _____

Signature 2: _____

Print your name NEATLY on the line below.

Name 1: _____

Name 2: _____

¹This is a shortened version of a problem in the eighth edition of *Advanced Engineering Mathematics* by Erwin Kreyszig

[**The Hypergeometric Equation:**] Gauss's hypergeometric differential equation is

$$x(1-x)y'' + [c - (a+b+1)x]y' - aby = 0. \quad (1)$$

In this equation a , b , and c are constants. We will see that by choosing appropriate values of these constants we can generate series representations for a wide variety of functions.

1. (5 pts) Assume that

$$y = \sum_{k=0}^{\infty} \alpha_k x^{k+r}$$

and use the generalized series method (the method of Frobenius) to show that either $r = 0$ or $r = 1 - c$.

2. (5 pts) Show that

$$\alpha_{n+1} = \frac{(n+a)(n+b)}{(n+1)(n+c)} \alpha_n.$$

3. (5 pts) Show that for $r = 0$ and $a_0 = 1$ the solution is

$$y(x) = 1 + \frac{ab}{1!c}x + \frac{a(a+1)b(b+1)}{2!c(c+1)}x^2 + \frac{a(a+1)(a+2)b(b+1)(b+2)}{3!c(c+1)(c+2)}x^3 + \dots \quad (2)$$

Explain why c cannot be a negative integer.

4. (3 pts) Some authors let $y(x) = F(a, b, c; x)$ to represent this function in order to easily indicate the values of the constants a , b , and c . Show that

$$F(1, 1, 1; x) = F(1, b, b; x) = F(a, 1, a; x) = \frac{1}{1-x}.$$

You will need to recall the Taylor series representation of $1/(1-x)$.

5. (2 pts) Show that if a or b is a negative integer then the series (2) is a polynomial.
6. (5 pts) Show that

$$\arctan x = xF\left(\frac{1}{2}, 1, \frac{3}{2}, -x^2\right).$$

You will need to find or compute the series representation of $\arctan x$.