PHY 230 Midterm 2 Takehome Problem

Rules:

1. You may use your books and your notes.
2. Unless otherwise noted, computers or calculators may be used to evaluate integrals.
3. Discussing this exam with members of other groups is not allowed.

Honor Pledge: On my honor, I pledge that I have not given, received, or tolerated others' use of unauthorized aid in completing this work.

Signature 1: ______________________________

Signature 2: ______________________________

Print your name NEATLY on the line below.

Name 1: ______________________________

Name 2: ______________________________

(25 pts) [**An Aging Spring**] In this project we will look at a model of an aging spring. We will use the series method to construct an approximate solution to this differential equation.

A differential equation describing an undamped mass-spring system with a decaying spring consant is

$$my'' + ke^{-\epsilon t}y = 0 \tag{1}$$

where $m > 0$ is the mass at the end of the spring.

1. *(4 pts)* What is the differential equation and its solution when $\epsilon = 0$? Explain the meaning of the $ke^{-\epsilon t}$ term in equation (1) when $\epsilon > 0$. In particular, explain why this models an aging spring.

2. *(2 pts)* Use the Taylor Series for e^x to write $e^{-\epsilon t}$ as a Taylor Series.

   Let's start solving this equation. For simplicity we will assume that $m/k = 1$ so we don't have to worry about these constants.

3. *(5 pts)* Assume that $y = \sum_{n=0}^{\infty} a_n t^n$ and show that

   $$\sum_{n=2}^{\infty} n(n-1)a_n t^{n-2} + \left[\sum_{n=0}^{\infty} \frac{(-\epsilon)^n}{n!} t^n\right]\left[\sum_{n=0}^{\infty} a_n t^n\right] = 0. \tag{2}$$

   Before we can use the series method for solving equation (1), we need to know how to multiply two infinite series together. Infinite series are mulitiplied together just like polynomials, only much longer. With a little work we get the formula

   $$\left(\sum_{n=0}^{\infty} a_n x^n\right)\left(\sum_{n=0}^{\infty} b_n x^n\right) = \sum_{n=0}^{\infty}\left[\sum_{k=0}^{n} a_k b_{n-k}\right] x^n. \tag{3}$$

4. *(6 pts)* Use reindexing and formula (3) to show that

   $$\sum_{m=0}^{\infty}\left[(m+2)(m+1)a_{m+2} + \sum_{k=0}^{m} \frac{(-\epsilon)^k}{k!} a_{m-k}\right] t^m = 0. \tag{4}$$

   Use this to find a recursion relationship (i.e. a formula for a_{n+2}.)

5. *(6 pts)* Use the initial conditions $y(0) = 1$ and $y'(0) = 0$ to compute $a_0, a_1, \ldots, a_6$. Write out $y(t)$ up to degree 6.

6. *(2 pts)* Let $\epsilon = 0$ in your answer above. Do you recognize this series? If so what is it and does it make sense?