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This week, we'll go over a few psychological strategies for problem solving such as wishful thinking and creativity. We'll go over a few problems from Chapter 2 of his book, *The Art and Craft of Problem Solving*. I'll discuss these three problems near the start of class:

1. Connect the boxes (page 17). Technique: Make the problem easier.
2. Monk climbing and descending the mountain. Technique: Bend the rules.
3. Integrated graph coloring. Technique: Maximize!

These are for you to work on. A hint for the last one: It's a classic application of the pigeonhole principle. If there are n pigeons in $n - 1$ holes, then at least one hole contains more than one pigeon. But what are the pigeons and what are the holes?

1. (Zeit 2.1.19) You are locked in a $50 \times 50 \times 50$ -foot room which sits on 100-foot stilts. There is an open window at the corner of the room, near the floor, with a strong hook cemented into the floor by the window. So if you had a 100-foot rope, you could tie one end to the hood and climb down to freedom. (The stilts are not accessible from the window.) There are two 50-foot lengths of rope each cemented to the ceiling about 1 foot apart, near the center of the ceiling. You are a strong, agile rope climber, good at tying knots, and you have a sharp knife. You have no other tools (not even clothes). The rope is strong enough to hold your weight, but not if it is cut lengthwise. You can survive a fall of no more than 10 feet. How do you get out alive?
2. (Zeit 2.1.21) A group of jealous professors are locked up in a room. There is nothing else in the room but pencils and one tiny scrap of paper per person. The professors want to determine their average (mean, not median) salary so that each one can gloat or grieve over their personal situation compared to their peers. However, they are secretive people and do not want to give away any personal salary information to anyone else. Can they determine the average salary in such a way that no professor can discover *any* fact about the salary of anyone but herself. For example, even facts such as "3 people earn more than \$40,000" or "no one earns more than \$90,000" are not allowed.
3. (Zeit 2.1.22) Bottle A contains a quart of milk, and bottle B contains a quart of black coffee. Take a tablespoon of the contents of A and mix it into B . Then take a tablespoon of the mixture in B and mix it into A . Which bottle has more impurities? I.e., is there more milk in B or coffee in A ?
4. The number $1000!$ ends with how many 0's?
5. (Larson, 3.2.1) Prove that any subset of 55 numbers chosen from the set $\{1, 2, 3, \dots, 100\}$ must contain two numbers differing by 9.