MCS-200 **In-class exercises** October 7, 2002
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This week's problems are from Loren Larson's book, *Problem-Solving Through Problems*. The book is superb at summarizing those techniques which come up in problem solving; it's excellent preparation for the Putnam!

This week's problems are primarily from two chapters: Heuristics (looking for a pattern) and Algebra. In class, I'll cover some material.

- Algebraic identities
- Unique factorization and GCD
- The Identity Theorem

(Repeat) Important Saturdays to mark on your calendar: November 9 is the ACM programming contest. November 16 is the PennePutnam team contest on-site. December 7 is the Putnam. February 22 is the Konhauser. The Mathematical Contest in Modeling will be held during a weekend in early February.

1. (Larson 4.1.1) Show that $n^4 - 20n^2 + 4$ is composite when n is any integer.
2. (Larson 4.2.2) Prove that the fraction $(n^3+2n)/(n^4+3n^2+1)$ is irreducible for every positive integer $n > 0$.
3. (Larson 4.3.1) Determine all polynomials $P(x)$ such that $P(x^2+1) = (P(x))^2+1$ and $P(0) = 0$.
4. (Larson 1.1.11) If $\langle a_n \rangle$ is a sequence such that for $n \geq 1$, $(2 - a_n)a_{n+1} = 1$, what happens to a_n as n tends toward infinity.
5. (Larson 1.1.6) Beginning with 2 and 7, the sequence $2, 7, 1, 4, 7, 4, 2, 8, \ldots$ is constructed by multiplying successive pairs of its members and adjoining the result as the next one or two members of the sequence depending upon whether the product is a one- or two-digit number. Prove that the digit 6 appears an infinite number of times in the sequence.
6. (Larson 1.1.4) Find positive numbers n and $a_1, a_2, \ldots, a_n$ such that $a_1 + a_2 + \cdots + a_n = 1000$ and the product $a_1 a_2 \cdots a_n$ is as large as possible.
7. (Will Shortz NPR Puzzle) This challenge comes from Erich Friedman, who contributed several puzzles to this year's World Puzzle Championship. Write out the digits from 1-9 in order. Then add some plus (+) signs and times (x) signs to the string to make it add up to 2,002. As usual in arithmetic, multiplication is done before addition, and you don't have to put a sign between every 2 digits. The answer is unique.