David Wolfe

1. (ARML 1991, T1) You have 121 marbles, some of which are red, some white, and the rest blue. You also have 10 jars. If all the marbles are distributed into the jars, there *must* be a jar with at least n marbles of the same color. Compute the maximum possible value for n.

2. (ARML 1991, T2) [Note: A palindrome is a positive integer that reads backwards the same as it reads forwards. For example: 67276.]

   The sum of two 4-digit palindromes is the 5-digit palindrome N. Compute the maximum possible value for N.

3. (ARML 1991, T3) Compute the smallest positive integer $n > 100$ such that $\binom{n}{101}$ is divisible by $\binom{n}{100}$, but is not equal to it.

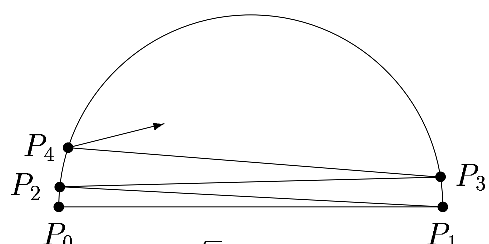
4. (ARML 1991, T4) If $(x^2 + x + 1)(x^6 + x^3 + 1) = \frac{10}{x-1}$, compute the real value of x.

5. (ARML 1991, T5) Compute the number of real values of x that satisfy the equation below. [Note: The vertical bars are absolute value signs.]

$$||x^2 - 1| - 1| = 2^x.$$

6. (ARML 1991, T6) One angle of a triangle is twice another, and the sides opposite these angles have lengths 15 and 9. Compute the length of the third side of the triangle.

7. (ARML 1991, T7) In the semicircle shown, diameter $P_0P_1 = 2$. Angle $P_0P_1P_2 = 1°$; angle $P_1P_2P_3 = 2°$; angle $P_2P_3P_4 = 3°$; ...; angle $P_{k-1}P_kP_{k+1} = k°$. If $\overline{P_kP_{k+1}}$ is the first chord whose length is less than 1, compute k.



8. (ARML 1991, T8) Let $x_n + iy_n = (1 + i\sqrt{3})^n$, where x_n and y_n are real and n is a positive integer. If $x_{19}y_{91} + x_{91}y_{19} = 2^k\sqrt{3}$, compute k.

9. (ARML 1991, T9) The bases on an isosceles trapezoid are 18 and 30, and its altitude is 8. Using each leg of the trapezoid as a diameter, semicricles are drawn exterior to the trapezoid. If the midpoints of those arcs are P and Q, compute PQ.

10. (ARML 1991, T10) Compute the smallest positive integer that can *not* be the difference between a square and a prime, if the square is greater than the prime.

One for home

11. (Larson 2.6.4) Prove that there exist integers a, b, c not all zero and each of absolute value less than one million, such that

$$|a + b\sqrt{2} + c\sqrt{3}| < 10^{-11}.$$

   Hint: Use the pigeonhole principle.