This week is a practice contest math competition. Each team has $1\frac{1}{2}$ hours to solve *and write up* solutions to the following five problems. Justifications are expected, not just the statement of an answer. Partial credit will be given for progress toward a solution or an answer to part of a question.

1. (NCS/MAA 1998 #1) Adolph and Bertha amused themselves for a while by matching pennies. On each toss, Adolph won a penny from Bertha if their coins matched, and Bertha won one from Adolph if they failed to match. When they stopped, their coins had matched 13 times and Bertha had gained 8 pennies. How many times did they toss? (Don't forget to explain your answer!)

2. (NCS/MAA 1998 #2) In base b, where $b > 9$, the quadratic equation $x^2 - mx + n = 0$ has roots 9 and 5. If the base b representation of m is 11, what is the base b representation of n?

3. (NCS/MAA 1997 #2) Prove that the sum of the cubes of three consecutive integers is always a multiple of 9.

4. (NCS/MAA 1998 #3) Recall that the hyperbolic functinos sinh and cosh are defined by

$$\sinh x = \frac{e^x - e^{-x}}{2} \text{ and } \cosh x = \frac{e^x + e^{-x}}{2}$$

   Suppose that

$$\sinh 2x + \cosh 2x = 2.$$

   Evaluate

$$\sinh 9x + \cosh 9x,$$

   and justify your answer.

5. (NCS/MAA 1997 #5) If

$$x = \frac{1 + \sqrt{1997}}{2}$$

   what is the value of

$$(4x^3 - 2000x - 1997)^{2003}?$$

   Justify your answer.

6. (NCS/MAA 1998 #6) A unit circle and 1998 distinct points P_i are chosen in a plane. Prove that there exists a point Q on the circle such that the sum of the distances from Q to the 1998 points P_i is greater than 1998.