MCS-200
David Wolfe

Putnam practice

December 2, 2002

Two numbers are relatively prime if they share no common factors. So 21 and 35 are not relatively prime (7 divides both), but 21 and 40 are relatively prime, since when we factor both $21 = 3 \cdot 7$ and $40 = 2 \cdot 2 \cdot 2 \cdot 5$, we find that they have no prime factors in common.

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1. (Putnam 1996 A-3) Suppose that each of twenty students has made a choice of anywhere from zero to six courses from a total of six courses offered. Prove or disprove: There are five students and two courses such that all five have chosen both courses or all five have chosen neither.

2. (Putnam 1997 A-2) Players $1, 2, 3, \ldots, n$ are seated around a table and each has a single penny. Player 1 passes a penny to Player 2, who then passes two pennies to Player 3. Player 3 then passes one penny to Player 4, who passes two pennies to Player 5, and so on, players alternately passing one penny or two to the next player who still has some pennies. A player who runs out of pennies drops out of the game and leaves the table. Find an infinite set of numbers n for which some player ends up with all n pennies.

3. (Putnam 1966 B-2) Prove that among 10 consecutive integers at least one is relatively prime to each of the others.

4. (Putnam 1978 A-1) Let A be any set of 20 distinct integers chosen from the arithmetic progression $\{1, 4, 7, \ldots, 100\}$. Prove that there must be two distinct integers in A whose sum is 104.