Homework set 1

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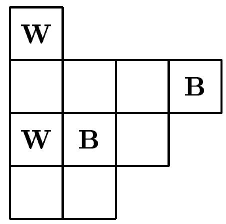
Due: September 12, 2002

1. (Edward Hordern) On the following bizarrely shaped chess board, there are two white knights and two black knights. You can move any of the four knights in one turn, so long as it remains on the board and doesn't land on another knight. (A chess knight can move from one square to another that differs by 1 in one coordinate and by 2 in the other coordinate.) Your goal is to exchange the Black and White knights in as few moves as possible.



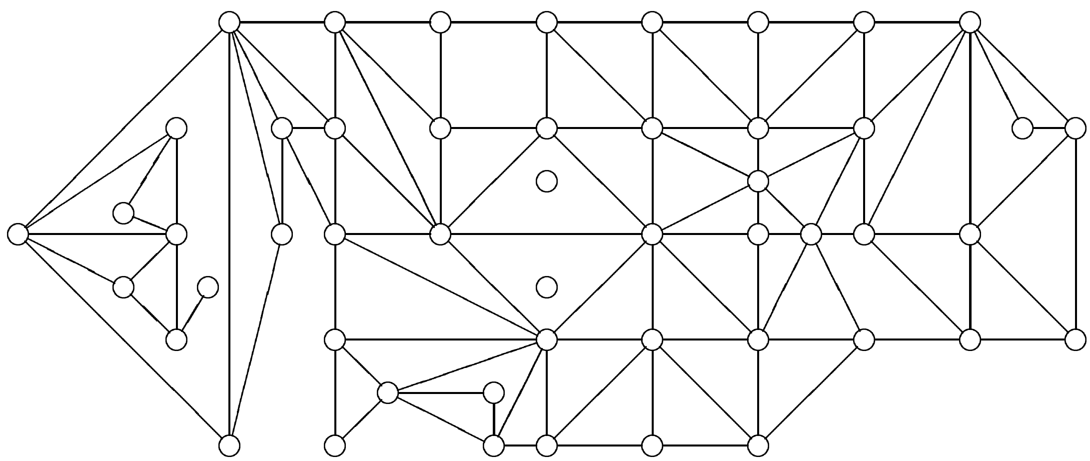
   (a) Identify the least number of moves required to exchange the two knights.

   (b) State your reasoning to convince the reader that, in fact, you've identified the minimum number of moves.

   Hint: Draw an appropriate graph and redraw it so it's easier to visualize.

2. (a) Where is Minnesota in the following graph G? Use graph theory vocabulary to explain your reasoning. Try to get as short a convincing argument as you can.

   (b) How many isomorphisms are there from G to itself?



3. (West 1.1.8) Prove that the 8-vertex graph on the left (1 3-dimensional cube) decomoposes into copies of $K_{1,3}$ and also into copies of P_4.

4. (Part of West 1.1.14) Prove that removing opposite corner squares from an 8×8 checkerboard leaves a subboard that cannot be partitioned into 1×2 and 2×1 rectangles. (Hint: A checkerboard can be 2-colored.)