

1. Complete the missing entries in the following table:

Decimal fractional notation	Binary positional notation
$-2\frac{1}{2}$	-10.1
23	10101
$\frac{1}{4}$	
$\frac{5}{8}$.01101
$-12\frac{9}{16}$	-11.10101
$\frac{1}{3}$	
$\frac{2}{5}$	11.0 $\overline{101}$

2. Consider the following 2-player subtraction game. There are a pile of n beans. A move consists of removing any proper factor of n beans from the pile. (For example, if there are $n = 12$ beans, you can leave a pile with 11, 10, 9, 8 or 6 beans.) The player to leave a pile with 1 bean wins.

Determine a winning strategy from those positions in which you can win. In particular, you'll need to determine which positions are "P-positions" (the Previous person to have moved should win), and which are "N-positions" (the Next person to move should win.) Prove your answer by induction.

How about the *misère* version? In *misère*, the player to leave a pile with 1 bean loses.