MCS-344 **Homework set 1 solution** February 10, 2003
David Wolfe Due: February 13, 2003

1. Complete the missing entries in the following table:

Decimal fractional notation	Binary positional notation
$-2\frac{1}{2}$	-10.1
23	$\boxed{10111}$
$\boxed{21}$	10101
$\frac{1}{4}$	$\boxed{.01}$
$\frac{5}{8}$	$\boxed{.101}$
$\boxed{\frac{13}{32}}$	.01101
$-12\frac{9}{16}$	$\boxed{-1100.1001}$
$\boxed{-3\frac{21}{32}}$	-11.10101
$\frac{1}{3}$	$\boxed{.\overline{01}}$
$\frac{2}{5}$	$\boxed{.\overline{0110}}$
$\boxed{3\frac{5}{14}}$	$11.0\overline{101}$

For the last entry, we can rewrite $.\overline{101}$ as:

$$\frac{5}{8} + \frac{5}{8}\left(\frac{1}{8}\right) + \frac{5}{8}\left(\frac{1}{8}\right)^2 + \frac{5}{8}\left(\frac{1}{8}\right)^3 + \cdots = \frac{5/8}{1-1/8} = 5/7$$

Dividing $.\overline{101}$ by 2 yields $.0\overline{101}$

2. Consider the following 2-player subtraction game. There are a pile of n beans. A move consists of removing any proper factor of n beans from the pile. (For example, if there are $n = 12$ beans, you can leave a pile with 11, 10, 9, 8 or 6 beans.) The player to leave a pile with 1 bean wins.

   Determine a winning strategy from those positions in which you can win. In particular, you'll need to determine which positions are "P-positions" (the Previous person to have moved should win), and which are "N-positions" (the Next person to move should win.) Prove your answer by induction.

   How about the *misère* version? In misère, the player to leave a pile with 1 bean loses.

A position is a P-position if and only if it has an odd number of beans. If there are an even number of beans, the player can win by taking away one bean, leaving an odd number (which, by induction, is a P-position.) If there are an odd number of beans, any move a player leaves an N-position, since odd numbers only have odd factors. Hence the position is an P-position. (Curiously, no base case is required in this proof. Why?) Surprisingly, the *misère* version is almost the same *except* a pile of size 2 is a P-position and a pile of size 3 is an N-position. To see this, use the same argument as above. Additionally note that neither player can leave a pile of size ≤ 2 unless the current pile is of size 3 or 4. Once the pile is of size 4, the winning strategy is to remove 2 beans instead of 1. For the base cases, confirm that size 2 is a P-position and size 3 is an N-position.