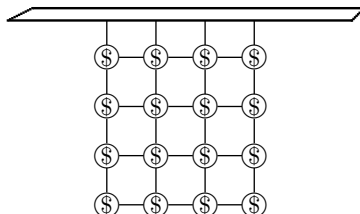


- Two players play the following game on a round table top of radius  $R$ . Players take turns placing pennies (of unit radius) on the tabletop, but no penny is allowed to touch another. Which of the two players has a winning strategy as a function of  $R$ ? Prove your answer by exhibiting the winning strategy.

The first player has a win so long as  $R \geq 1$ . The first player places a coin in the center of the table, and plays  $180^\circ$  symmetry thereafter.

- A bunch of coins is dangling from the ceiling. The coins are tied to one-another and to the ceiling by strings as pictured below. Players alternately cut strings, and a player whose cut causes any coins to drop to the ground loses. If both players play well, who wins? Prove your answer.



Let  $n$  be the number of coins, and  $s$  the number of strings. Each turn, a player must play on a cycle to keep coins from dropping. A graph consisting of the  $n + 1$  nodes ( $n$  coins and 1 ceiling) is connected and has no cycle if and only if it has  $n$  edges. (Apply, for instance, Euler's formula with 1 region, 1 connected component.) Hence the game lasts exactly  $n - s$  turns.

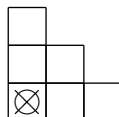
- The game of *Brussel Sprouts* starts with a number of 4-armed crosses on a piece of paper. A move is to draw a line from one arm to another arm. The line must not cross any other line or any cross. Once the line is drawn, 2 arms are drawn on the line, one on each side of the line. Determine who wins as a function of  $n$ , the number of initial crosses, and prove your answer. (Hint: how many moves does the game last?)

The first player wins if and only if  $n$  is odd.

Each move either connects two connected components, or divides a region in two. In either case, the resulting region(s) have at least one new arm. Hence, at the end of the game, each region has exactly one arm (for otherwise there would still be move(s) available). Also, the number of arms is constant, since each move removes two arms and adds two.

Now, if there are  $n$  brussel sprouts at the start of the game, then there are  $4n$  arms throughout the game, and the game ends when there are  $4n$  regions. Since each move either connects two components ( $n - 1$  moves) or creates a region ( $4n - 1$  new regions), we have a total of  $n - 1 + 4n - 1 = 5n - 2$  moves in the game.

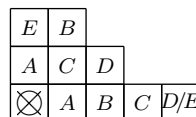
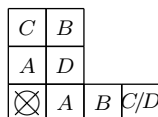
- Determine some of the  $\mathcal{P}$ -positions in 2-dimensional chomp. In particular, determine **all** the  $\mathcal{P}$ -positions for width 1 and width 2 boards, and find at least two  $\mathcal{P}$ -positions for boards that include the following 6 squares:



For width 1 boards, all boards are  $\mathcal{N}$ -positions except a single poison square, for in a single move the first player moves to just the single poison square.

A width 2 board is a  $\mathcal{P}$ -position if and only if the columns heights differ by 1. From such a  $\mathcal{P}$ -position, any move changes the difference in column heights. From an  $\mathcal{N}$ -position, it's easy to see that there is a move to a  $\mathcal{P}$ -position: If the columns are the same height, take one square, and if they differ by more than 1, reduce the left column to be 1 higher than the right.

The following positions are  $\mathcal{P}$ -positions. If one player moves on a square marked  $A$ , a move on the other square marked  $A$  is a winning response, vice-versa. (There are two responses to removing the lower-right square.)



5. Play several games of dots and boxes with a classmate or with a friend until you feel comfortable with the parity of the number of long chains. Now, play a *serious* game where you try to win and record the game, and submit the game record.