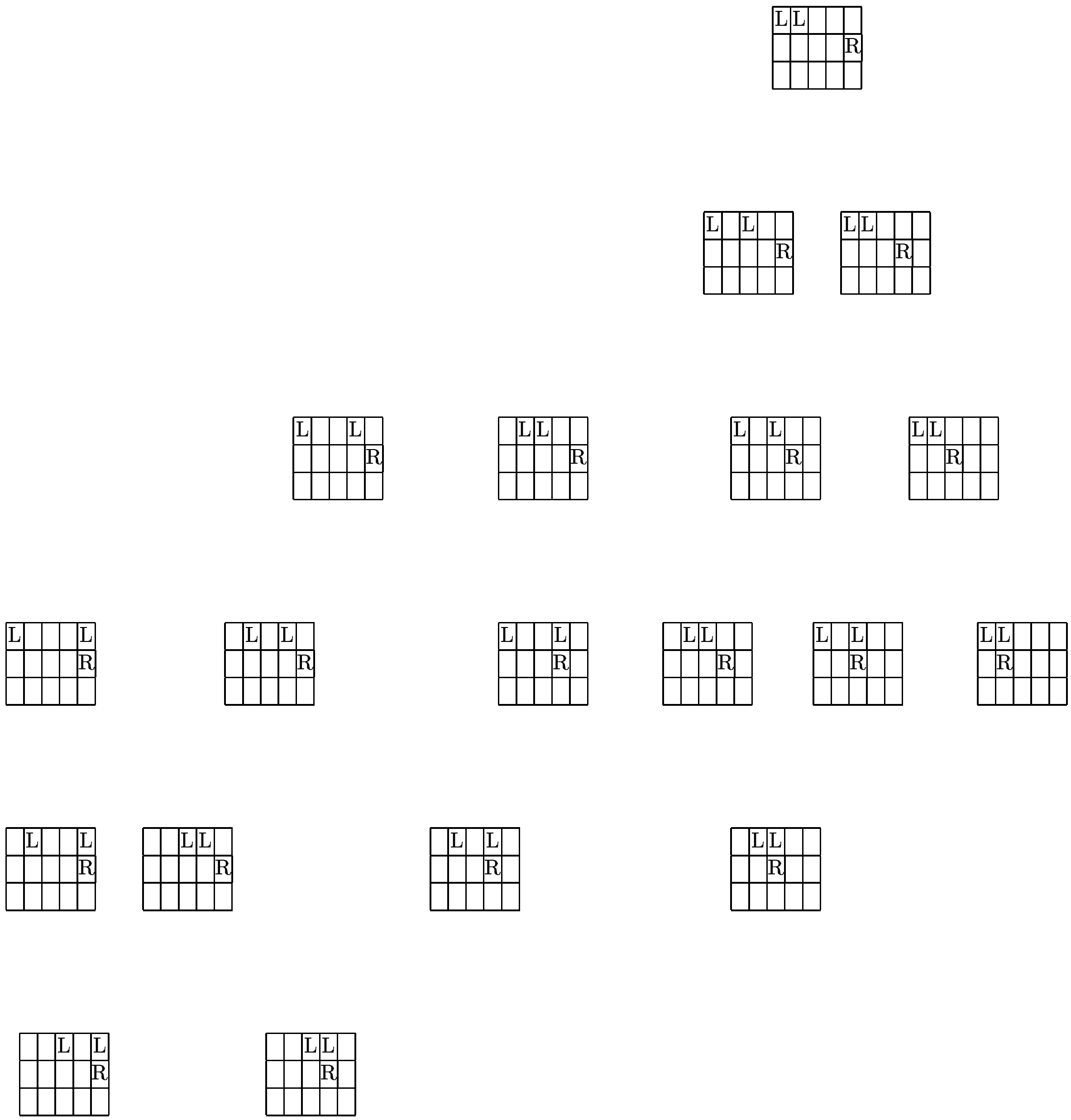
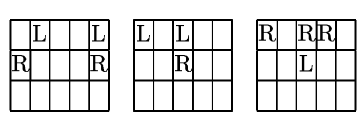
Homework set 5

1. We seek to evaluate ski-jump positions on the 3×5 board which have *two* Left skiers and one Right skier. If one of the skiers is already past the Right skier, the position should be easy to analyze without drawing a game tree. I have organized all other positions below so that you can fill in a game tree of all positions reachable from the top position. With the aid of the tables on pages 9-10 (old edition pages 11-12) of Winning Ways, you should be able evaluate all positions with imminent jumps, and then fill in the values of all remaining positions in the tree. (My apologies if I've made a typo or two.)

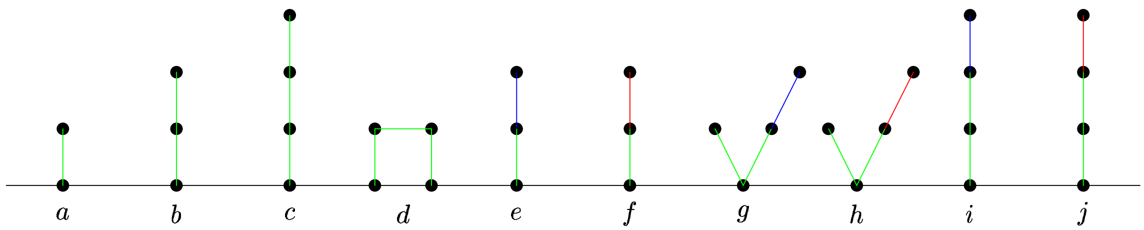


2. Who wins the following single ski-jumps position consisting of three slopes? Determine *all* winning moves.



3. Prove, algebraically, that games sums are commutative and associative. Work from the axiomatic definition of game sum,
$$G + H = \{\mathcal{G}^L + H, G + \mathcal{G}^L \mid \mathcal{G}^R + H, G + \mathcal{G}^R\}.$$
Clearly identify whenever induction is used.

4. Draw a partial order of the following Hackenbush positions:



5. (Optional) When given an open-ended question such as this, you should spend *at least* 3 hours on the homework set and report what you've found out. I hope that one of these sorts of problems will pique your interest enough that you'll want to investigate it further during the course.

   (a) Prove as much as you can about ski-jumps positions in which there are many Left skiers in the top row and one Right skier in the middle row. Such positions have been completely analyzed, but you may want to limit yourself to certain special cases. For example, what about the position in which neither player can play without creating an imminent threat? How about if there are only two Left skiers?

   (b) (Open problem) Analyze the games where Left has several skiers on the top and bottom rows versus Right's on the middle row of the $3 \times n$ board. Beware of the possibility that some potential jumps may be blocked by Left skiers on the bottom row. When there is only one Right skier and no Left skiers on the bottom row, then if the jump is not imminent, it happens that the value of the game is either an integer or half-integer. Is this still true under more general circumstances, with more skiers on the board?