Homework set 7 solution

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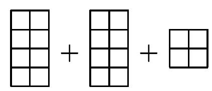
Due: March 27, 2003

You are free to use the computer to confirm calculations, so long as you try to gain confidence in your ability to do calculations by hand. Typically, it's wise to use the computer but *distrust it.* Take the time to make sure the answers it gives make sense to you! Show all work.

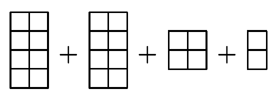
1. Convert the following to canonical form:
   (a) $5⧾_2$. By the number avoidance theorem, this is just $5||5|3$
   (b) $⧾_2 + ⧾_2 = 0|\{0,\{0|-2\} \mid -2\}$
   (c) $⧾_2 + ⧾_2 + ⧾_2 = ⧾_{2⧿_2}$
   (d) $⧾_2 + ⧾_3 = ⧾_2||⧾_2|-3⧾_2$
   (e) $⧾_2 + ⧿_2 = 0$
   (f) $⧾_2 + \uparrow = \uparrow||\uparrow|-2\uparrow$

2. For each of the following three domineering positions, evaluate the canonical form of the sum and determine whether you should play Left, Right, first, or second.

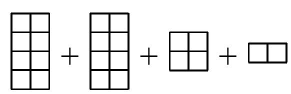
   (a)



   (b)



   (c)



   (a) The sum is $⧾_2 + ⧾_2 + 1|-1 = \{1 \mid -1⧾_2\}$. This is a win for the first player.
   (b) $2|⧾_2$ is a win for Left.
   (c) $\{0 \mid -2⧾_2\}$ is a win for the first player.

3. Fix a particular (partial) hackenbush position H, with one edge e unspecified. Four different positions can be obtained depending on whether edge e is blue, green, red or missing. (If missing, note that other edges might be disconnected from the ground and therefore removed as well.) Give the partial order on these four games, and prove that the partial order is independent of the position. As always, induction is prefered over arguments like, "and then right continues to..."

Denote the four games by G_+, G_*, G_-, and G_0, respectively. I assert that $G_+ > G_* > G_-$, and $G_+ > G_0 > G_-$, but that $G_* \parallel G_0$. By symmetry, it suffices to show $G_+ > G_*$, $G_+ > G_0$ and that $G_* \parallel G_0$.

- $G_+ - G_* > 0$: Left moving first can remove green edge e in $-G_*$, leaving $G_+ - G_0 > 0$ by induction. If Right removes e in $-G_*$, Right also loses by induction. For any other Right move, write can mirror the move in the other game, leaving some $G'_+ - G'_*$. If the move also removes e, the games are identical, and the difference is 0. If it does not, then $G'_+ - G'_* > 0$ by induction. Either way, Left wins.
- $G_+ - G_0 > 0$: Left moving first can remove the blue edge e in G_+ leaving $G_0 - G_0 = 0$. For Right's move, the argument is the same as the last case above.
- $G_* - G_0 \parallel 0$: Either player can win moving first by removing the green edge in G_* moving to $G_0 - G_0 = 0$.