MCS-375 **The Scorpion Problem** September 20, 2005
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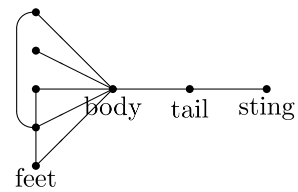
1. **(3.x — work alone)** A *scorpion* is a graph with three special nodes:



   (a) A *body* which is connected to all other vertices but the sting.

   (b) A *tail* connected only to the body and the sting

   (c) A *sting* connected only to the tail.

   All other vertices are called *feet*. The feet may be interconnected arbitrarily, and are connected to the body, but not the tail nor the sting. Given a scorpion on $n \geq 5$ nodes, find the sting in time $O(n)$ by asking questions of the form, "Is node u connected to node v?" (In other words, the graph is given as an adjacency matrix already loaded into memory.)

   Background: The significance of the scorpion graph is that it provides a counterexample to the conjecture, "Any non-trivial property of undirected graphs is evasive." A *property* of graphs must be invariant under isomorphism; in other words, it must be independent of the node-labels. Thus examples of non-trivial graph properties are: "G is connected" or "G is planar" or "G has 23 edges", while "Vertex A of G has degree 3" is not a graph property. A trivial graph property is one which is true (or false) for all graphs. A property is evasive if (in the worst case) all entries of an adjacencty matrix must be inspected in order to determine if a graph has the property. Virtually all graph properties you would naturally think of are evasive.