Fast Fourier Transform problems

1. This problem concerns the 8-input butterfly network FFT circuit from class.
   (a) Identify the pattern of inputs a_i down the left column. (Hint: Write the index i in binary.) Write down the order of the inputs for a 16-input FFT circuit.
   (b) After identifying the pattern of ω^k's on the edges, draw a 4-input butterfly network. (Hint: It's easiest to associate ω^k with the right endpoint of each arrow rather than the left endpoint.)
2. This problem asks you to compute the FFT and the inverse-FFT of that FFT on the vector $\overline{a} = (0, 1, 2, 3)$. I.e., $a_0 = 0$, $a_1 = 1$, $a_2 = 2$, and $a_3 = 3$. Recall that this represents the polynomial $A(x) = 3x^3 + 2x^2 + x$.
   (a) Draw a 4-input FFT circuit, leaving room at every node to be able to write a complex number like $2 - 4i$. What is ω equal to for this choice of $n = 4$?
   (b) Simulate the FFT circuit on $\overline{a} = (0, 1, 2, 3)$ by writing at each node the complex number which would be computed at that node. You should plug in the specific value of ω from part (a). Call the output vector $\overline{A} = (A_0, A_1, A_2, A_3)$. So, each A_i should be a specific complex number be equal to $A(\omega^i)$.
   (c) Check one of your answers, say A_3, by evaluating the polynomial $A(\omega^3)$ directly.
   (d) Now run the inverse FFT on the vector of complex numbers $\overline{A}$. Recall that the inverse FFT is identical to a normal FFT, only replace ω by $\omega^{-1} = \omega^{n-1}$. (Recall from class that you should get back the vector $a = (0, 1, 2, 3)$ with each entry multiplied by n. I.e., you should get back $0, 4, 8, 12$.)
   (e) Describe in your own words how 8-input FFT and inverse-FFT circuit(s) could be used to multiply two polynomials of degree 3.
3. Professor Pat Spacek is eager to multiply the polynomial x^{16} by x^{16} using the FFT algorithm. Pat mistakenly uses a 32-input FFTs (rather than 64-input FFTs) to do the calculations.
   (a) Why should Pat have used 64-input FFTs?
   (b) What did Pat's final answer come out to? (Hint: You don't need to draw a 32-input FFT circuit to find the answer! Understand the big picture at the top of CLRS page 828.)