MCS-385 **Homework set 3** March 21, 2003
David Wolfe Due: April 9, 2003

In this assignment, we'll be using `otter`. Otter is a Theorem prover developed by William McCune. A copy of the manual is in the lab, but I don't especially recommend you look at it at first. Instead, I recommend you simply mimic the examples in `steam.in`. To run it, type:

```
otter < ~wolfe/public/385/otter/examples/auto/steam.in
```

As you look at the file `steam.in`, and note note the following:

- Every sentence ends with a period. The error message isn't always clear about this so watch for it!
- Until you understand more about reasoning, your program should start `set(auto).` This tells `otter` to guess the best algorithms to use.
- Your *fact's* should be between "`formula_list(usable).`" and "`end_of_list.`". (Note that "`list(usable)`" has a different meaning. This would specify a list of *clauses* which should not use implication.)
- Proofs are by contradiction, so the last line has the negation of what we want to prove.
- "`all x`" quantifies over what is in the parentheses. For practical purposes, these parentheses are necessary.
- To get out of otter, type, "`kill.`" If you've typed some other junk before that, you may need to type it twice.
- `Emacs` has an otter mode. If you want to set this up, check the first few lines in my `.emacs` file. When editing an otter document (extension `.otter`), hit `C-h b` for bindings and C-h k to check out a particular key binding.

In general, debugging logic programs is painful work. (I'd say that this is a big problem with knowledge databases in general.) The reason for this is that it's very difficult to know if you have enough rules. Since I've spelled out all the rules you should need, debugging won't be as difficult here. You can **Debug each rule** in sequence: assert enough facts to bring it the rule into action and assert a contradiction. You can also prove a few things by hand, and try them out on `otter`. If you are frustrated, let me know, and I can tell you the answers to the challening questions; this will aid in your debugging.

You only need to use `otter` for the last of these problems, but I recommend you use it for all of them to get practice with it. In all cases, **transcribe the proofs into human-readable form** so that you are convinced that the proofs are valid and that you haven't mistakedly introduced a bug into one of the sentences.

1. (RN 5.17) We have assumed that the rules of each game define a utility function that is used by both players, and that a utility of x for MAX means a utility of $-x$ for MIN. Games with this property are called *zero-sum* games. (If you shift the utility function by a constant, the game is still called zero-sum.)

   (a) Most *real life* games (or situations) are non-zero-sum. Describe such a game. (For example, you could clearly explain why the current conflict between Bush and Hussein is not zero-sum.)

   (b) One might hope that some variant of alpha-beta pruning could still be used for non-zero-sum games. Prove that *no* pruning *at all* is possible (including alpha-beta pruning!) when you have non-zero sum games.

   To aid in your intuition, you could draw game trees such as in Figure 5.6 on page 131, but the leaf nodes would have values like $3 : -3$ or $3 : -2$ to mean one player gets 3 points and the other gets -2 points. If all nodes are of the form $x : -x$, it's a zero-sum game. Again, the point is to explain why no pruning at all is possible.

2. In this assignment you will play the role of a detective. Your goal is to solve a murder mystery using your knowledge of logic and inference. Last night, someone was murdered. The body has not been identified yet; therefore, you will have to identify both the victim and the murderer. The home where the victim was assaulted is a simple four-room building. The four rooms are the dining room, the library, the kitchen, and the conservatory. Figure 2 illustrates the relative position of each room. Passages between adjacent rooms are indicated by the four open doorways in the diagram.

   The four murder suspects are Mrs. White, Ms. Scarlet, Mr. Green, and Colonel Mustard. One of these four people was murdered by one of the others. The killer used one of the following four weapons to commit the crime: a revolver, a lead pipe, a rope, or a knife.

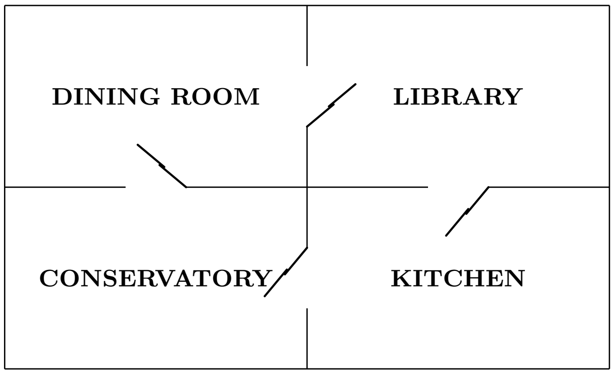
   Assume that we do not know the location of the body within the house. However, we know that the body produces a stench in the room that it is in, and the two rooms that are connected to it via a doorway. For this Problem, you are to write your answers using propositional symbols of the following forms:

   ST-i: There is a Stench in room i where i is one of the four rooms L, K, D, C . Example: ST-L means that there is a stench in the Library.

   Body-j: The body is in room j where j is one of the four rooms. Example: Body-C means that the body is in the Conservatory.

   (a) Using only the propositions ST-i and Body-j, write a set of propositional logic sentences that express the following

      i. If there is a stench in a particular room, then the body must be either in that room or in one of the two connecting rooms.



ii. If there is no stench in a particular room, then the body is not in that room or either of the two connecting rooms.

Please number the sentences you write.

(b) Assume that you know the following:

- The body is not in the Dining Room.
- There is a stench in the Library.
- There is a stench in the Conservatory.

Express these clues in propositional logic. Using your answer from Part (a), construct a propositional logic proof that reveals the location of the body. You may add any additional propositional sentences necessary to capture knowledge about the problem. However, you are restricted to using only the ST-i and Body-j propositional symbols. Furthermore, you must provide an explanation for why you included the additional sentences. To specify your proof:

- Number each of your propositional sentences as a separate rule. Examples: R1, R2, R3, ...
- Identify the inference rule that you used to generate each new propositional sentence. Example: Modus Ponens, And-Elimination, etc. (See p. 172 of AIMA for names of the inference rules. You may use rules from 6.2 on page 180 to simplify expressions without referencing them. See also `~wolfe/public/385/unicorn.in` for a proof of the unicorn problem.)
- List the rules that you used to construct each new propositional sentence. Example: (R1 + R17).

(c) (Optional — I recommend doing this if you need more practice before moving on to the challenging part.)

In addition to the four weapons found in the house, police caught one of the four suspects carrying a candlestick. Assume that the following sentences are true for this candlestick:

- The candlestick is valuable if it is shiny.
- If the candlestick is either lethal or a trinket, then it is shiny.
- If the candlestick is a weapon, then it is lethal; however, if it is not a weapon, then it is a harmless trinket.

Can you prove whether or not the candlestick is:

i. A weapon?
ii. Valuable?
iii. Shiny?

Use the following notation:

SH: The candlestick is shiny.
VA: The candlestick is valuable.
WE: The candlestick is a weapon.
TR: The candlestick is a trinket.
HA: The candlestick is harmless.
LE: The candlestick is lethal.

3. It turns out that the body was found in the Kitchen. The coroner has established that the murder took place between 6pm and Midnight last night. However, the problem here is that the four suspects may have been in different rooms at each of these times. Solving the murder may depend on finding a good way to express knowledge about where the suspects were at various times.

   For this Problem, you are restricted to using propositional symbols of the form

   In(i, j, k)**:** Suspect j was in room i at time k, where k is one of $\{6, 12\}$. Example: **In**(D,W,6) means that Mrs. White was in the Dining Room at 6.

   V(i**):** Suspect i is the victim. Example: V(W) means that Mrs. White is the murder victim.

   (a) Describe a way of expressing the following clue in propositional logic. Feel free to describe the pattern of sentences schematically, rather than listing all of them explicitly. How many propositional logic sentences does your formulation require? Explain why this many sentences are needed.

   If a suspect is not the victim, then the suspect was in a different room at each of the two times.

   (b) Show how to express the clue of Part A using first-order logic (FOL). Provide a clear explanation of your predicates. How many FOL sentences are required?

   (c) Consider the following sentences

   - Every suspect who was alive at midnight was also alive at 6pm.
   - The murderer is one of the suspects who was alive at midnight.
   - The victim did not change rooms after being murdered.

   Which of the two logics, i.e. propositional logic vs. FOL, is better for expressing these three sentences? Assume that we prefer to write as few logical sentences as possible. Explain the ontological commitments (see the textbook definition) that make this logic the better choice for writing these sentences. Finally, write these three sentences in the logic that you chose.

4. For this problem, use the following notation to refer to the rooms, suspects, and weapons. Again, you must use `otter` and send me your code. The proofs take seconds for `otter` to find, so if `otter` takes longer suspect a bug.

   Rooms

   L: Library
   K: Kitchen
   D: Dining Room
   C: Conservatory

   Suspects

   W: Mrs. White
   S: Ms. Scarlet
   G: Mr. Green
   M: Colonel Mustard

   Weapons

   RE : Revolver
   RO : Rope
   KN : Knife
   PI : Lead Pipe

   You have an old crime-solving agent to help you in your investigation. However, this agent only understands sentences that are written in predicate logic. Therefore, before your agent can help you solve the crime you must describe the crime scene, suspects, and weapons using FOL. You should use the following predicates:

   Suspect(i)**:** Object i is a suspect.

   Weapon(i)**:** Object i is a weapon.

   Murderer(i)**:** Object i is the murderer.

   MWeapon(i)**:** Object i is the murder weapon.

   InRoom(j, i, t)**:** Object j is in room i at time t.

Victim(i): Object i is the murder victim.

(a) Describe the crime scene, suspects, and weapons in first-order logic. Furthermore, only consider the times 6pm and midnight. Each suspect may be in a different room at each of these times. However, a particular suspect can only move into a room that is connected to their starting location by a door. Weapons cannot be moved from room-to-room. Assume that if two suspects are in the same room at the same time, then they can see each other and any weapon that is in that room. Similarly, if a suspect is in the same room with the victim, then the suspect can see the victim.

Express the following clues using FOL:

i. One of the suspects is the murderer.
ii. One of the suspects is the victim.
iii. The crime was not a suicide.
iv. The murder took place at either 6pm or midnight.
v. If a suspect is in the same room with the victim at the same time, then the suspect is the murderer. (Hint: You'll also want to add rules that say all 4 suspects are, in fact, different people.)
vi. If two suspects are in the same room at the same time, then they can see each other.
vii. The dead suspect is the victim.
viii. Any weapon that is in the same room with the victim is the murder weapon.
ix. Mr. Green was in the same room as the knife at 6pm, and was in the Conservatory at midnight.
x. Knives are always kept in the kitchen.
xi. Ms Scarlet was never in the same room with any other suspect.
xii. Colonel Mustard always goes into the Kitchen at 6pm.
xiii. Mrs. White left the Dining Room at 6pm, but did not go into the Conservatory.
xiv. If a suspect saw Colonel Mustard at 6pm, then Colonel Mustard was dead at 6pm.
xv. Nothing can be in two places at one time.

(b) Using your answer above, answer the following questions using first-order inference. You will have to add additional sentences to those given in above to solve this case. However, clearly explain your reason for adding each additional sentence. Answer the following questions.

i. Who was the murderer?
ii. Who was the victim?
iii. What was the murder weapon?
iv. In which room did the murder take place? (You'll need another axiom to define the MurderRoom.)

5. (In groups) Our goal in this problem is to do a feasability study to determine whether `otter` is an appropriate theorem proving program to implement a Wumpus world agent. Since `otter` runs *off-line*, you'll need to rebuild the database after every step the Agent makes.

There are two feasibility issues you will investigate:

(a) Is `otter` expressive enough to encode everything required to build a knowledge base?

(b) Is `otter` fast enough to respond to a query from the knowledge base? How large a world do you expect `otter` will be able to handle given your experiments?

You should produce a report which thoroughly addresses both of these issues. You will need to include specific expriments you performed, and, when appropriate, timing results of those experiments and discuss how this impacts the feasibility of using `otter` for the agent's knowledge base. Be sure to test each piece of the sample knowledge base rules as you go! (Since `otter`'s source code is publicly available, you could in principle, design a Wumpus world in C.)