MAA Summer Seminar **Games day 1** August 12, 2003

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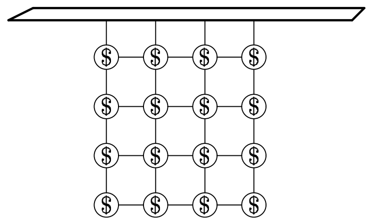
1. Two players play a vertex deletion game on an undirected graph. A turn consists of removing exactly one vertex of even degree (and all edges incident to it.) Determine under what conditions the first player has a winning strategy and determine the strategy.

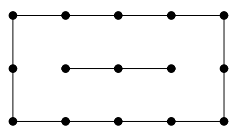
2. A bunch of coins is dangling from the ceiling. The coins are tied to one-another and to the ceiling by strings as pictured below. Players alternately cut strings, and a player whose cut causes any coins to drop to the ground loses. If both players play well, who wins? Prove your answer.

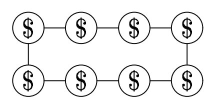


3. The game of *Brussel Sprouts* starts with a number of 4-armed crosses on a piece of paper. A move is to draw a line from one arm to another arm. The line must not cross any other line or any cross. Once the line is drawn, 2 arms are drawn on the line, one on each side of the line. Determine who wins as a function of n, the number of inital crosses, and prove your answer. (Hint: how many moves does the game last?)

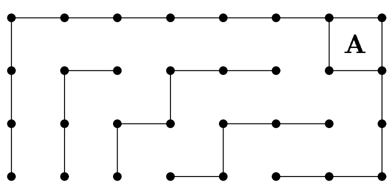
4. Two players play the following game on a round table top of radius R. Players take turns placing pennies (of unit radius) on the tabletop, but no penny is allowed to touch another. Which of the two players has a winning strategy as a function of R? Prove your answer by exhibiting the winning strategy.

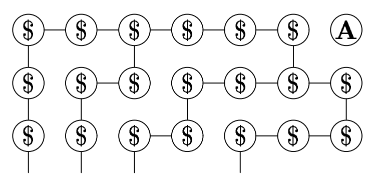
5. There are other Dots & Boxes (or Strings & Coins) positions where every move is loony. For example, there can be cycles:





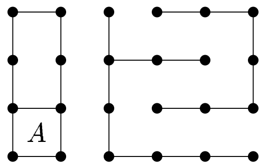
and many-legged *spiders*:

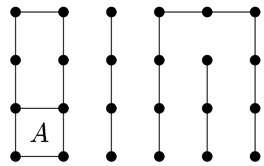


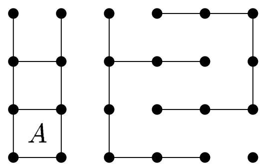


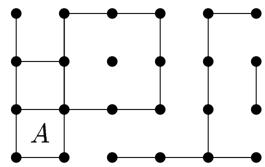
How can you adapt the Long Chains Theorem and its proof to account for these positions?

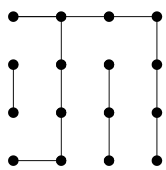
6. In each of the following Dots & Boxes positions, you are A and your opponent is B. It is your turn. Find a winning move if you can. If there are none, write *resign*. Be prepared with a brief explanation. If the score looks close, be sure winning on the long chains is enough!

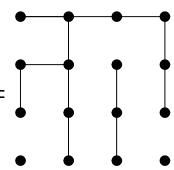
$A =$ 

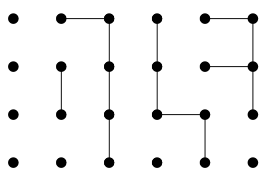
$B =$ 

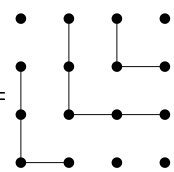
$C =$ 

$D =$ 

$E =$ 

$F =$ 

$G =$ 

$H =$ 

$I =$