MAA Summer Seminar **CGSuite Exercises** August 12, 2003

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1. Let G be the game $\{2||1|*\}$. Calculate the canonical form of G, $G+G$, and $G+G+G+G$.

2. How do $\uparrow$ and $\Uparrow$ compare with $*$? How do they compare with $*n$ for $n > 1$?

3. Use the LeftOptions and RightOptions commands to determine the canonical form of $n.\uparrow$ for various n. ($n.\uparrow$ is the game $\uparrow + \uparrow + \cdots + (n \text{ times}) + \cdots + \uparrow$.)

4. Find the canonical form H of a 4×4 Domineering rectangle. Try comparing H with small positive and negative numbers. Make a conjecture as to whether or not H is an infinitesimal, and then use IsInfinitesimal(H) to verify it. How does H compare with various tinies?

5. Try calculating the canonical form of $\{a|b\}+*$ for various numbers a, b with $a > b$. Make a conjecture regarding the general form.

6. Calculate the canonical forms of the following Clobber positions and observe that a pattern occurs:

7. Try calculating the canonical form of $\boldsymbol{+}_a + \boldsymbol{+}_b$ for various numbers a, b with $0 < a < b$. Make a conjecture regarding the general form $G_{a,b}$ of such sums.

8. If you have time (and after you've finished exercise 6), try to find constraints on positive numbers a, b, c $(a > b)$ such that

$$\{a|b\} + \boldsymbol{+}_c = \{a + \boldsymbol{+}_c | b + \boldsymbol{+}_c\}.$$