Games problems

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1. Consider the following 2-player subtraction game. There are several piles of beans. Each player takes turn moving on one of the piles. A move on a pile of size n consists of removing any proper factor of n beans from the pile. (For example, if there are $n = 12$ beans in a pile, you can leave the pile with 11, 10, 9, 8 or 6 beans.) It's not legal to take the last bean in a pile. The last player to move wins.

   (a) Determine a rule for computing the nim-value of a single pile. Can you prove your rule always works?

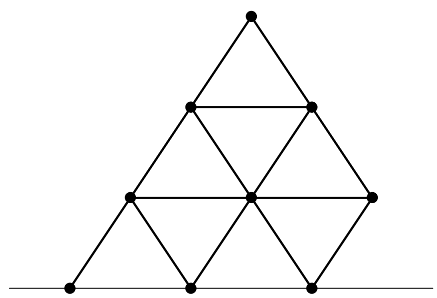
   (b) Use your rule to find all winning moves in the four pile game with piles of sizes 8, 10, 12 and 16.

   (c) What happens if any you can remove any factor (not just *proper factors*), so that it's now legal to take a whole pile?

2. Fix a particular (partial) hackenbush position G, with one edge e unspecified. Four different positions can be obtained depending on whether edge e is blue, green, red or missing. (If missing, note that other edges might be disconnected from the ground and therefore removed as well.) Give the partial order on these four games, and prove that the partial order is independent of the position. As always, induction is prefered over arguments like, "and then right continues to..."

3. Prove that any red-blue hackenbush position is a number.

4. Here is a not-yet-colored Hackenbush graph with 14 branches:



   (a) What is the value of the graph if it is colored all blue?

   (b) What is the value of the graph if it is colored all green?

   (c) Using red, blue, and green colors however you wish, color the branches to make the value of this graph the smallest positive number that you can.

   (d) the largest infinitesimal that you can.

   (e) the positive infinitesimal of smallest atomic weight that you can.

   You need not find the optimal answers.