Distinguishing Gamblers
from Investors
at the Blackjack Table

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OUTLINE

• A little history
• Blackjack rules and about card-counting
• Evaluating an individual’s strategy
• Evaluating expected winnings for a baseline strategy
• Unresolved issues
HISTORY

- (1956) Baldwin, Cantey, Maisel, McDermott: Basic strategy
- (1966) Thorp, *Beat the Dealer*
- About blackjack card-counters
BLACKJACK RULES

- Play for 21: hit, stand, bust, hard, soft, deck, shoe
- Fixed dealer strategy
- Blackjack pays 3:2
- Double down
- Split
- Variations: decks, surrender, insurance, resplitting, double after split, playing multiple hands, ...
## Basic strategy for an infinite deck

<table>
<thead>
<tr>
<th>HARD</th>
<th>SOFT</th>
<th>SPLIT</th>
</tr>
</thead>
<tbody>
<tr>
<td>4–8 HHHHHHHHHH</td>
<td>2 HHHHHHHHHH</td>
<td>A PPPPPPPPPP</td>
</tr>
<tr>
<td>9 HHDDDDHHHH</td>
<td>3 HHHHDDHHH</td>
<td>2 HPPPPPHHH</td>
</tr>
<tr>
<td>10 HDDDDDDHHH</td>
<td>4 HHHHDDHHH</td>
<td>3 HPPPPPHHH</td>
</tr>
<tr>
<td>11 HDDDDDDDDH</td>
<td>5 HHHHDDHHH</td>
<td>4 HHHHPPHHH</td>
</tr>
<tr>
<td>12 HHHSSHHHH</td>
<td>6 HHHHDDHHH</td>
<td>5 HDDDDDDDDH</td>
</tr>
<tr>
<td>13 HSSSSSHHHH</td>
<td>7 HHHHDDHHH</td>
<td>6 HPPPPPHHH</td>
</tr>
<tr>
<td>14 HSSSSSHHH</td>
<td>8 SSDDDSSHH</td>
<td>7 HPPPPPHHH</td>
</tr>
<tr>
<td>15 HSSSSSHHH</td>
<td>9 SSSSSSSSSS</td>
<td>8 PPPPPPPPPP</td>
</tr>
<tr>
<td>16 HSSSSSHHH</td>
<td>10 SSSSSSSSS</td>
<td>9 SPPPPSPPS</td>
</tr>
<tr>
<td>17–21 SSSSSSSSS</td>
<td>11 SSSSSSSSS</td>
<td>T SSSSSSSSSS</td>
</tr>
</tbody>
</table>
CARD COUNTING

• A typical counting system maintains

\[
\frac{(\text{number of } 10\text{'s and } A\text{'s}) - (\text{number of } 2\text{'s through } 6\text{'s})}{(\text{number of decks remaining})}
\]

• Effect on play

• Effect on wagers

• Basic strategy gives typically gives dealer .5% advantage

• Counting well typically gives player .5% advantage
GOAL and MOTIVATION

Evaluate a player’s skill, measured by long term expected winnings

• To help gamblers
• Larger question of evaluating decisions under uncertainty
CHALLENGES

- The problems of variance
- No knowledge of player strategy
- Can’t expose player to all possible situations
- Computational efficiency
KEY IDEA

1. Determine and evaluate baseline strategy
2. Compare expectation of player’s choices to baseline
3. Credit or debit player’s expected winnings rather than actual
Define

\[ C \overset{\text{def}}{=} \text{Sample space of cards remaining} \]
\[ O \overset{\text{def}}{=} \text{Sample space specifying ordering of the deck} \]
\[ W_S \overset{\text{def}}{=} \text{Winnings playing strategy } S \]
\[ B \overset{\text{def}}{=} \text{Any automated baseline strategy} \]

Then,

\[ E_C \{ E_O \{ W_S \} \} = E_C \{ E_O \{ W_B \} \} \]
\[ + E_C \{ E_O \{ W_S \} - E_O \{ W_B \} \} \]

- \( E_O \{ W \} \) has variance exceeding 1.3 (with unit bets).
- \( E_O \{ W_S \} - E_O \{ W_B \} \) typically has variance between \( 10^{-6} \) and \( 10^{-5} \).
\[
C \stackrel{\text{def}}{=} \text{Sample space of cards remaining}
\]
\[
D \stackrel{\text{def}}{=} \text{Sample space specifying ordering of initial deals}
\]
\[
O \stackrel{\text{def}}{=} \text{Sample space specifying ordering of the deck after deal}
\]
\[
W_{SS'} \stackrel{\text{def}}{=} \text{Winnings wagering strategy } S, \text{ playing strategy } S'
\]
\[
B \stackrel{\text{def}}{=} \text{Any automated baseline strategy}
\]
\[
\mathbb{E}_C \left\{ \mathbb{E}_D \left\{ \mathbb{E}_O \{W_{SS}\}\right\}\right\} = \mathbb{E}_C \left\{ \mathbb{E}_D \{\mathbb{E}_O \{W_{SB}\}\} \right. \\
\left.+ \mathbb{E}_D \{\mathbb{E}_O \{W_{SS}\} - \mathbb{E}_O \{W_{SB}\}\}\right\}
\]
\[
\bullet \mathbb{E}_D \{\mathbb{E}_O \{W_{SB}\}\} \text{ typically has variance between } 10^{-3} \text{ and } 10^{-4}.
\]
HIGH-LOW COUNT SYSTEMS

- Count \((Tens + Aces - 2s - 3s - 4s - 5s - 6s)\)
- Balanced count system
- Count per deck affects both play and wager strategy
- Wagers vary from 1 unit to 10 units
- Dubner 1963, Wong 1994
- Wong predicts expected winnings of \(.020 \pm .002\) units per hand

\[
S = \text{Wong’s strategy} \\
B = \text{semi-perfect strategy}
\]
### SIMULATION RESULTS

<table>
<thead>
<tr>
<th>Hands Simulated</th>
<th>Average Actual Winnings</th>
<th>Hands</th>
<th>Assessed Expectation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Run 1</td>
<td>Run 2</td>
<td>Run 1</td>
</tr>
<tr>
<td>10,000</td>
<td>-.06128</td>
<td>.05743</td>
<td>100</td>
</tr>
<tr>
<td>20,000</td>
<td>-.00184</td>
<td>.05600</td>
<td>200</td>
</tr>
<tr>
<td>50,000</td>
<td>.04133</td>
<td>.02950</td>
<td>500</td>
</tr>
<tr>
<td>100,000</td>
<td>.03622</td>
<td>.01845</td>
<td>1,000</td>
</tr>
<tr>
<td>200,000</td>
<td>.02756</td>
<td>.03175</td>
<td>2,000</td>
</tr>
<tr>
<td>500,000</td>
<td>.02317</td>
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<td>5,000</td>
</tr>
<tr>
<td>1,000,000</td>
<td>.02221</td>
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<td>10,000</td>
</tr>
<tr>
<td>2,000,000</td>
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<td>.02262</td>
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</tr>
<tr>
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<td>.02197</td>
<td>.02059</td>
<td>50,000</td>
</tr>
<tr>
<td>10,000,000</td>
<td>.01873</td>
<td>.02002</td>
<td>100,000</td>
</tr>
<tr>
<td>Variance</td>
<td>20</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Difference of about .04 due to approximations of EV's
UNRESOLVED ISSUES

- Assessing risk: $E \{\log(\$)\}$
- Why not change the distribution to assess?
- Playing to affect the deck
- Real casino: deck isn’t random, multiple players, multiple hands, etc.