Calculus I: Optimization

First set up, but do not solve the following problems. In other words, find a function of one variable with an appropriate domain that you would find the maximum or minimum of in order to solve the problem. You should be able to pass your set-up to another student to solve as a calculus optimization problem. Finally, solve the problems.

1. The sum of two positive numbers is 48. What is the smallest possible value for the sum of their squares?

2. I want to make a fish tank with a volume of $2\text{m}^3$ and I would like the base of the tank to be a rectangle twice as long as it is wide. The base and sides of the tank are to be of glass. What shape of tank will use the least amount of glass (and so cost least)?

3. A rectangular storage container with an open top is to have a volume of $10\text{m}^3$. The length of its base is twice the width. Material for the base costs $10$ per square meter. Material for the sides costs $6$ per square meter. Find the cost of materials for the cheapest such container.

4. Gustavus is building a new running track. It is to be the perimeter of a region obtained by putting two semicircles on the ends of a rectangle. However, owing to financial constraints, the administration has decided to grow corn in the area surrounded by the track. If the track is to be 440 yards long, determine the necessary dimensions to build the track in order to maximize the area for growing corn.
5. A farmer is planning to plant a small orchard, and is gathering information about the amount of fruit he can expect to harvest each year once the trees mature. He is advised that, if he plants up to 60 trees of a particular type on his plot of land the average harvest from each tree will be about 120 kg, but for each additional tree planted the expected yield will go down by an average of 2 kg per tree as a result of overcrowding. Naturally, he wants to plant for the maximum yield of fruit. How many trees should he plant?

6. A feedlot farmer has 200 feet of fencing with which to enclose two adjacent rectangular corrals. What dimensions should be used so that the enclosed area will be a maximum?

7. A wire 2 meters long is cut into two pieces. One piece is bent into a square for a stained glass frame while the other piece is bent into a circle for a TV antenna. To cut down on storage space, where should the wire be cut to minimize the total area of both figures? Where should the wire be cut to maximize the total area?

8. A wire 6 meters long is cut into twelve pieces. These pieces are welded together at right angles to form the frame of a box with a square base. Where should the cuts be made to maximize the volume of the box? Where should the cuts be made to maximize the total surface area for the box?