MCS 256 Discrete Calculus and Probability  Set 3

Homework rules

- Acknowledge your sources (people and texts).
- In nontrivial problems, show how you get your answers.
- Turn in neat, well-written solutions, not messy first drafts. Trim “fringes.”
- Do not copy collaborative solutions; write up solutions in your own words.
- Turn in homework on time. Each class day late reduces the possible points by 25%.

Problems

1. Calculate the Stirling numbers of the second kind \( \binom{n}{k} \) for \( 0 \leq k \leq n \leq 8 \).

2. Calculate the Stirling numbers of the first kind \( \left\{ \begin{array}{c} n \\ k \end{array} \right\} \) for \( 0 \leq k \leq n \leq 8 \).

3. Complete this table with first, second, third, fourth, and fifth forward differences of\[ f(x) := g(2x) \]

<table>
<thead>
<tr>
<th>x</th>
<th>g(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-8</td>
<td>829</td>
</tr>
<tr>
<td>-6</td>
<td>407</td>
</tr>
<tr>
<td>-4</td>
<td>161</td>
</tr>
<tr>
<td>-2</td>
<td>43</td>
</tr>
<tr>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>-1</td>
</tr>
<tr>
<td>4</td>
<td>-23</td>
</tr>
<tr>
<td>6</td>
<td>-109</td>
</tr>
<tr>
<td>8</td>
<td>-307</td>
</tr>
</tbody>
</table>

4. Complete the table of divided differences for the function \( g(x) \) in the previous problems. Determine the most likely values of \( g(10) \) and \( g(12) \)

5. You are given that \( f(0) = 3, f(3) = 18, \) and \( f(5) = 48 \).
   - (a) Estimate \( f(1) \) using divided differences.
   - (b) Find the polynomial \( p(x) \) of degree 2 that agrees with \( f(x) \) at \( x = 0, 3, 5 \).

6. “An interpolation for \( f(4) \) is based upon the following values:
   \( f(0) = 5, f(3) = 7, f(5) = 10, f(15) = 50 \).
   After the computations have been performed, it is discovered that \( f(5) \) should have been 12. Find the error in the interpolated value of \( f(4) \).” [Kellison 120]

References