MCS 118
Worksheet on graphs

1. Sketch a rough graph of each of the following power functions and then check your work on a calculator:

   (a) $y = -5x^3$

   (b) $y = \frac{x^4}{2}$

   (c) $y = .0004x^4$

   (d) $y = x^3$

   (e) $y = -4x^2$

   (f) $y = -3x^7$

2. When you graph a polynomial, the big picture will look like the graph of the leading term of the polynomial. Use your calculator to graph each of the following polynomials for $-500 \leq x \leq 500$. Compare these graphs to the first four graphs in the previous problem.

   (a) $y = 3x^2 - 5x^3 + 10$

   (b) $y = \dfrac{x^4 - 20x^2}{2}$

   (c) $y = .0004x^4 + 0.2x^2 - 0.3x - 1$

   (d) $y = x^3 - 1.5 \cdot x^2 - 4.5x + 5.0$

3. However, when you graph polynomials in a small window, the graphs may cross the x-axis several times, and go up and down in between. For example, graph the following polynomials for $-5 \leq x \leq 5$.

   (a) $y = 3x^2 - 5x^3 + 10$

   (b) $y = \dfrac{x^4 - 20x^2}{2}$

   (c) $y = .0004x^4 + 0.2x^2 - 0.3x - 1$

   (d) $y = x^3 - 1.5 \cdot x^2 - 4.5x + 5.0$

4. The number of x-intercepts of a graph will be at most the degree of the polynomial; the number of bumps will be at most one less than the degree. For example, graph the following polynomials. Change the size of the window until you can clearly see the x-intercepts.

   (a) $y = 0.1x^3 - x$

   (b) $y = x^3 - x + 3$

   (c) $y = x^4 - 2x^2 - 5$

5. How the graph behaves at an x-intercept depends on how many times the factor divides the polynomial. More precisely, if $p(x) = (x-a)^n \cdot q(x)$, where $q(x)$ is another polynomial that doesn't have $(x-a)$ as a factor, then the graph bounces off (or kisses) the x-axis at the point $(a, 0)$ when n is even. When n is odd, the graph goes on through the x-axis. For example, graph the following polynomials.

   (a) $y = (x-1)^2(x+1)$

   (b) $y = (2x+3)^3(x-3)^2$

6. Find the equation of the line in the graph below.