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Chemical Storage

Problem We are given n compounds numbered from 1 to n. Each compound j must be stored at a constant temperature lying in some closed interval $[L[j], U[j]]$. We want to assign every compound j to a temperature t_j in its interval, using the fewest number of distinct temperatures possible. All compounds assigned the same temperature will be stored in the same refrigerator. This means we are minimizing the number of refrigerators.

Algorithm

1. Sort the compounds into nondecreasing order using their upper bound as key.
Now we have compounds $j = 1, \ldots, n$ with lower bound storing temperature at $L[j]$, upper bound storing temperature at $U[j]$, with $U[1] \le U[2] \le \cdots \le U[n-1] \le U[n]$.
2. Find the minimum number of refrigerators k and storing temperatures $T[1..n]$ for all chemicals as follows.

```
k ← 0
/* we maintain variable u equal to the upper bound of the last refrigerator alloted */
u ← −∞
for j ← 1 to n do {
    if L[j] > u then {
        u ← U[j]
        k ← k + 1
    }
    T[j] ← u
}
```

Correctness

To prove correctnesss, we show that the number of fridges k returned by the algorithm is a lower bound for any algorithm that solves the problem. For $1 \le j \le k$, let I_j be the interval that causes our algorithm to start a new fridge. These k intervals are mutually non-overlapping. Therefore, no two compounds associated with those intervals can be assigned to the same fridge! So any valid solution requires $\geq k$ fridges. Our algorithm returns exactly k fridges, so it must be optimal.

Running Time

Step 1 uses time $O(n \log n)$ using heap sort, for exaple.
Step 2 uses time $O(n)$ since we process each compound in time $O(1)$.

Remark

This problem is the Minimum Clique Cover Problem on interval graphs. Even though the Minimum Clique Cover Problem on general graphs is NP-complete, this handout shows it is solvable in polynomial time when restricted to interval graphs.