MCS 321 Midterm Exam 1 Notes and Theorems
Spring 2010

Below is a list of some important things that you need to know for the first exam. This list is **not** comprehensive.

Properties and Operations of Complex This includes properties of $|z|$, $\overline{z}$, addition, multiplication, etc. Don't forget about the argument and principal argument.

Euler's Formula You should know Euler's Formula $e^{i\theta} = cos\theta + i\sin\theta$ and the polar representation of complex numbers $z = re^{i\theta}$. You should know DeMoivre's Formula and understand how it relates to Euler's Formula. You should be able to convert from rectangular to polar representation and vice versa (for numbers with arguments you can fairly easily determine).

Roots You should be able to compute integer roots of complex numbers.

Complex Functions You should be able to perform basic manipulations with complex functions such as finding their real part (u) and imaginary part (v). You should be able to describe the geometry of the types of functions we discussed.

Complex Exponential and Trig Functions You should know the definitions of these functions.

$$e^z = e^x e^{iy},\ \sin z = \frac{e^{iz} - e^{-iz}}{2i},\ \cos z = \frac{e^{iz} + e^{-iz}}{2}$$

You should be able to do basic operations with them and use the identities to prove basic properties of these functions.

Complex Log A branch of the log function with range $y_0 \leq \mathrm{Im}(\log z) < y_0 + 2\pi$ is defined by

$$\log z = \log|z| + i\arg z.$$

Limits and continuity You should know the definition of the limit. In particular, you should understand how it is similar to the definition for real-valued functions and how it is different. You should be able to compute limits or show that one does not exist.

Definition of the Derivative $f'(z_0) = \dfrac{f(z) - f(z_o)}{z - z_0}$

Cauchy-Riemann Equations If $f(z)$ is analytic at $z = z_0$ and $f = u + iv$ then

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \text{ and } \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

Here is a list of theorems from class you should be able to prove. You should also review your homework for likely suspects. Note I've shortened the statements of these theorems for ease of typing. You should be sure that you know the proper statement.

1. **Cauchy-Riemann** (first half) If f is complex differentiable then it satisfies the Cauchy-Riemann equations.
2. If $f(z) = u(x, y) + iv(x, y)$ then both u and v are harmonic.
3. If f is analytic on a domain D and $f'(z) = 0$ on D then f is constant on D.
4. If f and $\overline{f}$ are both analytic on a domain D then f is constant on D.
5. The Binomial Theorem.